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BRILL-NOETHER THEORY FOR STABLE VECTOR BUNDLES

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§0. Introduction. Let C be a projective nonsingular curve of genus g. Given integers n, d, there is a moduli space $S'(n, d)$ parametrizing stable vector bundles of rank n and degree d on C. When n, d are not coprime, $S'(n, d)$ is not compact. It may be compactified to a scheme $S(n, d)$ by adding equivalence classes of semistable vector bundles. Recall that a vector bundle is stable (respectively semistable) when for every proper subbundle F

$$\mu(F) = (\deg F/\operatorname{rank} F) < \mu(E) \qquad (\text{respectively} \leqslant).$$

Any semistable vector bundle E has a filtration $0 = E_0 \subset E_1 \subset \cdots \subset E_k = E$ such that $\mu(E_i/E_{i-1}) = \mu(E)$ and E_i/E_{i-1} is stable. The vector bundle $\oplus(E_i/E_{i-1})$ is then uniquely determined by E and is called the graduate of E. Two vector bundles on C are said to be equivalent when their graduates are isomorphic (see for instance [S]).

Given n, d and an integer r, let $W'^{r}_{n,d}$ be the sublocus of $S'(n, d)$ consisting of those stable vector bundles with at least $r + 1$ independent sections. Define also $W^{r}_{n,d}$ as the sublocus of $S(n, d)$ with a representative possessing at least $r + 1$ independent sections.

Consider the following problems. What is the dimension of $W'^{r}_{n,d}$ (resp. $W^{r}_{n,d}$) when C is a generic curve in the moduli space $\mathscr{M}_g$ of curves of genus g? What is its singular locus?.

From the theory of determinantal varieties (see for instance [A, C, G, H] Ch.II), one would expect their dimension to be

$$\rho(n, d, r, g) = n^2(g - 1) + 1 - (r + 1)(r + 1 - d + n(g - 1))$$

where a negative value of $\rho(n, d, r, g)$ should mean that they are empty.

Also

$$\operatorname{Sing}(W'^{r}_{n,d}) = W'^{r+1}_{n,d}.$$

These results are in fact true when $n = 1$. However, for $n \geqslant 2$ one cannot expect such a satisfactory answer. For instance, it is possible that $\rho \geqslant 0$ for negative values of the degree, while no semistable vector bundle of negative degree may have

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