Vol. 62, No. 1 DUKE MATHEMATICAL JOURNAL © February 1991

L^2 BOUNDEDNESS OF OSCILLATORY INTEGRAL OPERATORS

YIBIAO PAN

1. Introduction. In this paper, we give a solution to a problem about the L^2 boundedness of certain oscillatory integral operators, which was proposed by D. H. Phong and E. M. Stein in [PS].

The operators we study here are of the form

$$(Tf)(x) = \int_{R^n} e^{i(Bx,y)} K(x-y) f(y)\, dy \tag{1.1}$$

where (Bx, y) is a real bilinear form, and $rank(B) = k$, K is a function which is smooth away from the origin, homogeneous of degree $-(n-k)$.

Problem. When are the operators in (1.1) bounded operators on $L^2(R^n)$?

Here is some background of this problem. This type of operator originated from the study of the singular Radon transform in the model case by Phong and Stein. For operators

$$f \to p.v. \int_{R^n} e^{i(Bx,y)} K(x-y) f(y)\, dy, \tag{1.2}$$

where K is C^∞ away from the origin, coincides with a homogeneous function of degree $-\mu$ for large $|x|$, with a homogeneous function of degree $-n$ for small $|x|$, and satisfies the cancellation condition

$$\int_{|x|=\varepsilon} K(x)\, d\sigma(x) = 0 \tag{1.3}$$

for ε small, Phong and Stein showed that if $\mu > n - rank(B)$, then these operators are bounded on $L^2(R^n)$.

Clearly, in the problem mentioned above, the kernel functions are homogeneous of critical degree, i.e., $\mu = n - rank(B)$. In fact, when $rank(B) = 0$ ($\mu = n$), these operators are simply the classical singular integral operators, by the theorem of

Received 6 February 1990. Revision received 12 May 1990.
The author was supported in part by a Sloan Doctoral Fellowship.