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GENERIC UNIQUENESS FOR AN INVERSE BOUNDARY VALUE PROBLEM

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0. Introduction and statement of results. Electrical impedance tomography is concerned with determining the spatially dependent conductivity of a body Ω from steady state direct current measurements at the boundary. This problem arose in geophysics in determining the conductivity of the earth at depth from surface measurements. More recently it has been proposed as a valuable diagnostic tool in medicine and biology as a noninvasive method to determine conductivity contrasts in the human body. Calderón [C] formulated the general n-dimensional problem and obtained the first results. We describe now in more detail the mathematical problem.

Let $\Omega \subset \mathbb{R}^n$, $n \geqslant 2$, be a bounded, smooth domain and $\gamma \in L^\infty(\bar{\Omega})$, $\gamma \geqslant \varepsilon > 0$ in $\bar{\Omega}$, be the isotropic conductivity of Ω. If a voltage potential $f \in C^\infty(\partial\Omega)$ is applied on the boundary, then under the assumption of no sinks or sources of current the resulting potential u in Ω satisfies the Dirichlet problem:

$$\begin{cases} L_\gamma u = \operatorname{div}(\gamma \nabla u) = 0 & \text{in } \Omega \\ u|_{\partial\Omega} = f. \end{cases} \tag{0.1}$$

The voltage to current (or Dirichlet to Neumann) map is then defined by

$$\Lambda_\gamma \colon f \to \gamma \frac{\partial u}{\partial \nu}\bigg|_{\partial\Omega}, \tag{0.2}$$

where u is the solution of (0.1) and ν the outer normal on $\partial\Omega$. Significant progress has been made since Calderón's pioneering paper studying the uniqueness question for the map

$$\Phi \colon \gamma \to \Lambda_\gamma. \tag{0.3}$$

Kohn and Vogelius settled the question of uniqueness in the real analytic [K-V, I] and piecewise real analytic category [K-V, II]. Sylvester and Uhlmann [S-U, I] proved uniqueness in the C^∞ category in dimensions $n \geqslant 3$. The smoothness assumption can be relaxed to $\gamma \in W^{2,\infty}(\Omega)$ as a consequence of the result of Nachmann, Sylvester and Uhlmann [N-S-U]. Chanillo [Ch] extended this further, particularly

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