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THE LINEARIZATION OF HIGHER CHOW CYCLES OF DIMENSION ONE

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1. Introduction. In [1] S. Bloch defined higher Chow groups of codimension p cycles for any quasi-projective scheme over some base scheme S which, for an affine scheme X, are given by the homology of the complex

$$\xrightarrow{\partial} z^p(X, i) \xrightarrow{\partial} z^p(X, i-1) \xrightarrow{\partial} \cdots \xrightarrow{\partial} z^p(X, 1) \xrightarrow{\partial} z^p(X, 0),$$

where $z^p(X, i)$ is the free abelian group generated by codimension p subvarieties of $X \times \Delta^i$ which intersect all the faces Δ^{i-1} transversally and the differential given by the alternating sum of intersections. Bloch's complex $z^p(X, *)$ contains a subcomplex generated by all linear varieties, the so-called affine Grassmannian Homology complex ${}^A\mathrm{CG}^p_*$ [7]. Beilinson, MacPherson and Schechtman conjectured [3] that the inclusion of chain complexes ${}^A\mathrm{CG}^p_* \hookrightarrow \mathbf{Z}^p(\mathit{Spec}\ k, *)$ between the affine Grassmannian complex and Bloch's complex induces an isomorphism (up to torsion) of homology groups

$$pr_*\colon {}^A\mathrm{GH}^p_q \to \mathrm{CH}^p(\mathit{Speck}, p+q). \tag{1}$$

It turns out that this conjecture is not true. In fact a counterexample is given for codimension 2 cycles on Δ^3. The main object of this article is the investigation of the map pr_* of (1). Its significance lies in the role it plays in understanding the γ-filtration quotients of algebraic K-theory tensored with the rationals. As one already knows from work of Suslin [11] and Bloch [1] and an explicit calculation of the Grassmannian Homology groups [7], the map in (1) is a rational isomorphism for some particular choices of the parameters p and q. In fact, for all $p \geqslant 0$, we have

$$pr_*\colon GH^p_0(k) \otimes \mathbf{Q} \xrightarrow{\cong} \mathrm{CH}^p(\mathit{Spec}\ k, p) \otimes \mathbf{Q} \cong K^M_p(k) \otimes \mathbf{Q} \cong gr^p_\gamma K_p(k) \otimes \mathbf{Q}.$$

The main result of the current chapter is

THEOREM 4.2. *For all $p \geqslant 1$ the map*

$$pr_*\colon {}^A\mathrm{GH}^p_1 \otimes \mathbf{Q} \to \mathrm{CH}^p(\mathit{Spec}\ k, p+1) \otimes \mathbf{Q}$$

is a surjection.

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