Vol. 62, No. 1 DUKE MATHEMATICAL JOURNAL © February 1991

DISTINGUISHED p-ADIC REPRESENTATIONS

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§1. Introduction. Let E/F be a quadratic extension of number fields, and M a quaternion algebra over F. Let H and G be the multiplicative groups of M and $M \otimes_F E$, respectively, thought of as the F-rational points of two F-groups. The "relative trace formula" describes the integrals

$$I(f) = \int\int K^f_{\text{cusp}}(x, y)\, dx\, dy, \qquad x, y \in Z_H(F_{\mathbb{A}})H(F)\backslash H(F_{\mathbb{A}}),$$

where K^f_{cusp} is the cuspidal kernel associated to a Hecke function f on $G(F_{\mathbb{A}})$ and Z_H is the center of H. (See [5].) For a suitable finite set S of places of F, this is expressed as a linear combination of characters

$$(*) \qquad I(f) = \sum_{\pi} a_\pi \chi_\pi(f^S),$$

where $f^S = \otimes_{v \notin S} f_v$ and π ranges over the automorphic, cuspidal representations of $G(F_{\mathbb{A}})$, with trivial central character, which are "unramified outside S." (Additional assumptions on f assure that the functions f^S comprise a commutative algebra.) Comparisons involving the relative trace formula for two such algebras M_1 and M_2 come down to comparing two expressions of the form (*) and equating coefficients.

If f can be chosen so that the coefficient a_π is nonzero, then π is said to be a *distinguished representation.* Equivalently, there exists a smooth function φ in the space of π such that $B(\varphi) \neq 0$ where

$$B(\varphi) = \int_{Z_H(F_{\mathbb{A}})H(F)\backslash H(F_{\mathbb{A}})} \varphi(h)\, dh.$$

It has been shown in [4] and [8] that an automorphic, cuspidal representation π of $GL(2, E_{\mathbb{A}})$ is distinguished precisely when it is the base change lift of an automorphic cuspidal representation of $GL(2, F_{\mathbb{A}})$ whose central character is the quadratic idele class character of F attached to E.

The original motivation for distinguished representations can be found in [4], where the authors investigate the poles of the Hasse-Weil functions attached to a certain Shimura surface. A formula of Brylinski-Labesse is used to express the zeta function in terms of Eisenstein integrals which have poles with residue $B(\varphi)$. The

Received 18 November 1989. Revision received 2 April 1990.