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CORRECTION TO WHITTAKER MODULES ASSOCIATED WITH HIGHEST WEIGHT MODULES

HISAYOSI MATUMOTO

On p. 83, an argument listed as a proof of Theorem 3.2.6 is not sufficient to prove the theorem. It only proves that $\mathrm{Dim}(U(\mathfrak{g})\mathrm{Wh}_\Psi(L_\Psi(\mu))) \leqslant \mathrm{Dim}(L(\mu))$ for $\mu \in \mathbf{P}_S^{++}$. Unless Ψ is admissible, I do not know whether $U(\mathfrak{g})\mathrm{Wh}_\Psi(L_\Psi(\mu)) = L_\Psi(\mu)$ holds or not. At this time, I do not have a counterexample or a proof of the statement of 3.2.6.

Corollary 3.2.8 and Proposition 3.2.9 depend on Theorem 3.2.6. However, they follow from the following weaker version of Theorem 3.2.6, which we can prove.

THEOREM. *Let* $\mu \in \mathbf{P}_S^{++}$ *and* $\mathrm{Dim}(L(\mu)) = d_\Psi$. *Then* $L_\Psi(\mu)$ *is a finitely generated* $U(\mathfrak{g})$*-module and we have* $\mathrm{Dim}(L_\Psi(\mu)) \leqslant d_\Psi$.

Proof. Assume $\mu \in \mathbf{P}_S^{++}$ and $\mathrm{Dim}(L(\mu)) = d_\Psi$. As on p. 83 line 13, we have:

$$\dim F_\Psi^n(V^*) \leqslant \frac{c}{d_\Psi!} n^{d_\Psi} + O(n^{d_\Psi - 1}) \qquad (n \to \infty). \tag{$*$}$$

Here, $V = \bar{L}(-\mu)$, c is the multiplicity of $L(\mu)$, F_Ψ^n is defined on p. 71. First, we prove $L_\Psi(\mu)$ is finitely generated. We assume $L_\Psi(\mu)$ is not finitely generated, hence there exists some strictly increasing sequence of finitely generated $U(\mathfrak{g})$-submodules $\{L_n | n \in \mathbb{N}\}$.

Since $\bar{L}_k = L_{k+1}/L_k$ is nontrivial, the multiplicty $c(\bar{L}_k)$ is nonzero. This means $c(L_k) \geqslant k$ for all k. From Proposition 2.4.3, we have the following statement which clearly contradicts $(*)$.

$$\dim F_\Psi^{d_S n}(V^*) \geqslant \frac{k}{d_\Psi!} n^{d_\Psi} - O(n^{d_\Psi - 1}) \qquad (n \to \infty), \tag{$**$}$$

for all $k \in \mathbb{N}$. Here, d_S is a constant defined on p. 72.

Now that we see $L_\Psi(\mu)$ is finitely generated, there exists some n such that $F_\Psi^n(L_\Psi(\mu))$ generates $L_\Psi(\mu)$. Since $U_i(\mathfrak{g})F_\Psi^n(L_\Psi(\mu)) \subseteq F_\Psi^{n+d_S i}(L_\Psi(\mu))$, the theorem follows from $(*)$. □

REFERENCES

1. HISAYOSI MATUMOTO, *Whittaker modules associated with highest weight modules*, Duke Math. J. **60** (1990), 59–113.

MATHEMATICAL SCIENCES RESEARCH INSTITUTE, 1000 CENTENNIAL DRIVE, BERKELEY, CALIFORNIA 94720

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