Vol. 61, No. 3 DUKE MATHEMATICAL JOURNAL © December 1990

SEPARATING TOPOLOGY AND NUMBER THEORY IN THE ATIYAH-SINGER g-SIGNATURE FORMULA

DANIEL BEREND AND GABRIEL KATZ

0. Introduction. One of the most useful results in equivariant differential topology is the celebrated Atiyah-Singer g-signature formula [AS]. In this paper we present a new version of this formula, "separating" its number theoretical and topological ingredients. This enables us to clarify the geometrical meaning of the latter ones and, at the same time, makes the interplay between the topology of smooth cyclic actions and the number theory of the corresponding cyclotomic fields more transparent. For other interactions of this sort see [HZ]. Moreover, we improve (Proposition 2.1 and Theorem 2.2) the g-signature theorem [AS] by proving some integrality results concerning its topological ingredients, the so-called normal quasi-signatures (see Section 2, especially (2.5) and (2.6), for definitions).

A distinguishing feature of our approach is that the number-theoretical aspects of the g-signature formula depend only on the representation normal to the fixed point set of the action, and not on the totality of the normal bundles. On the other hand, we show that the topological complexity of the bundles normal to the g-fixed point set contributes to the g-signature only via some special integers $\{S_{\vec{\omega}}\}$—the (normal) quasi-signatures of these bundles (see Theorem 2.2). Related ideas, in the special case of semifree cyclic actions with real codimension 2 fixed point set, may be found in [H2, Section 4, Theorem]. Although the derivation of our formula is based on the original Atiyah-Singer theorem, the new version may suggest an alternate, more geometrical, proof of this theorem. Such a proof may, in turn, be extended to a broader category of actions, not necessarily smooth.

Let us introduce a few notations. C_m is the cyclic group of order m, and g an arbitrary fixed generator thereof. C_m acts on an even-dimensional, closed manifold M by orientation-preserving diffeomorphisms. Denote by M^g the set of C_m-fixed points in M. Throughout the paper, it is assumed that the normal bundle $\nu(M^g, M)$ admits an equivariant complex structure (for odd m this assumption is automatically fulfilled). Denote by $\boldsymbol{\mu}$ an isomorphism class of a typical complex C_m-representation normal to M^g. Let $M^g_{\boldsymbol{\mu}}$ be the components of M^g having $\boldsymbol{\mu}$ as the complex slice-type of their generic point. $Sign(g, M)$ stands for the g-signature of M [AS]. This invariant lies in the ring of integers $\mathbf{Z}[\lambda]$ of the cyclotomic field $\mathbf{Q}(\lambda)$, $\lambda = \exp(2\pi i/m)$.

We shall now describe the plan of the paper. In Section 1 we introduce a notion of g-signature for equivariant, symmetric or antisymmetric, unimodular bilinear

Received August 31, 1988. Revision received March 27, 1990. Research partially supported by the Basic Research Foundation of the Israel Academy of Sciences and Humanities, grant No. 111/87, and by the United States-Israel Binational Science Foundation, grant No. 86-00208.