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GREEN'S CURRENTS AND HEIGHT PAIRING ON COMPLEX TORI

JEAN-BENOÎT BOST

In this paper, we prove the following theorem, which extends a result of Bloch and Hain (cf. [BH] and §4).

THEOREM 0.1. *Let A be a complex torus of dimension n and let Z_1 and Z_2 be two analytic cycles in A, of respective codimensions p_1 and p_2, such that $p_1 + p_2 = n + 1$ and $|Z_1| \cap |Z_2| = \emptyset$. Let $Z_1 \boxminus Z_2$ be the divisor in A defined as the direct image of the cycle $Z_1 \times Z_2$ in $A \times A$ by the map*

$$A \times A \to A$$

$$(x, y) \mapsto x - y.$$

Then we have

$$\langle Z_1, Z_2 \rangle_\infty = \langle \{0\}, Z_1 \boxminus Z_2 \rangle_\infty. \tag{0.1}$$

The bracket $\langle\ ,\ \rangle_\infty$ denotes the height pairing between cycles "at infinite places" introduced by Gillet and Soulé ([GS1]) and by Beilinson ([Be2]). This archimedean height pairing depends *a priori* on the choice of a Kähler metric on A. Here, we take any translation invariant metric on A; the height pairing does not depend on the chosen invariant metric.

The archimedean height pairing $\langle Z_1, Z_2 \rangle_\infty$ was first introduced by Bloch and by Beilinson ([B12], [Be1]) for *homologically trivial cycles*. Then its definition does not require any additional structure, such as a Kähler metric, on the ambient variety. Theorem 0.1 shows that even when one is concerned only by the pairing between homologically trivial cycles—for instance, when one deals with the Bloch-Beilinson conjectures—the extended definition may be relevant.

We will give two proofs of Theorem 0.1. First, it will be obtained as a consequence of the following statement, which is analogous to the well-known formula of "reduction to the diagonal" in ordinary intersection theory (see also [B12], equation (3.11)).

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