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BIEXTENSIONS AND HEIGHTS ASSOCIATED TO CURVES OF ODD GENUS

RICHARD HAIN

1. Introduction. Suppose that $\mathscr{X}^{n+1}$ is a regular, projective scheme over spec $\mathbb{Z}$. One can think of $\mathscr{X}$ as being a family of n-dimensional projective varieties over the curve spec $\mathbb{Z}$: the fiber X_p of $\pi\colon \mathscr{X} \to \operatorname{spec} \mathbb{Z}$ over the prime ideal (p) is simply the reduction of $\mathscr{X}$ modulo p. The base spec $\mathbb{Z}$ can be compactified by adding a point ∞ corresponding to the imbedding of $\mathbb{Z}$ into $\mathbb{C}$. The family π is then compactified by adding the complex points X_∞ of $\mathscr{X}$, an n-dimensional complex projective manifold, over ∞.

Likewise, two algebraic cycles $\mathscr{Z}$, $\mathscr{W}$, flat over spec $\mathbb{Z}$ of respective dimensions $d+1$ and $e+1$, can be thought of as families of cycles $Z_p \subseteq X_p$ and $W_p \subseteq X_p$ of dimensions d and e respectively. To simplify matters, we assume that their generic fibers Z_∞ and W_∞ are homologous to zero in X_∞ and have disjoint supports.

If $d + e = n - 1$, then $\mathscr{Z}$ and $\mathscr{W}$ are of complementary dimension in $\mathscr{X}$. In this case their *Arakelov intersection number*, or *height pairing*

$$\langle \mathscr{Z}, \mathscr{W} \rangle \in \mathbb{R}$$

is defined ([A1], [A2], [B1], [Be], [GS], [G]). This mysterious number decomposes as a sum

$$\langle Z_\infty, W_\infty \rangle_\infty + \langle \mathscr{Z}, \mathscr{W} \rangle_{fte}.$$

Heuristically, the finite part $\langle \mathscr{Z}, \mathscr{W} \rangle_{fte}$ can be considered as a sum

$$\sum_{\substack{p \in \mathbb{N} \\ \text{prime}}} \langle Z_p, W_p \rangle_p$$

where the contribution at the prime p is

$$\langle Z_p, W_p \rangle_p = \log p \times (\#\text{ points in } Z_p \cap W_p, \text{ counting multiplicities})$$

when $\mathscr{X}$ has good reduction at p. The archimedean contribution is given by the

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