Vol. 61, No. 3 DUKE MATHEMATICAL JOURNAL © December 1990

NUMERICALLY POSITIVE LINE BUNDLES ON AN ARAKELOV VARIETY

MINHYONG KIM

In this paper, we prove a variation on the "arithmetic ampleness" result of Gillet-Soulé [8], giving a numerical criterion for the existence of arithmetic sections of high powers of a metrized line bundle on an Arakelov variety. Our result differs from those of Faltings [4] and Gillet-Soulé in that many sections of small norm that are *linearly independent on the generic fibre* are shown to exist, assuming numerical positivity.

In the first part, we prove, for concreteness, our result on an arithmetic surface. This allows, furthermore, for easy comparison with an entirely similar result for a fibred algebraic surface over a field proved in the second part, stressing again "the analogy between function fields and number fields" in the spirit of Weil [17]. (This section was suggested by B. Mazur.)

Our theorem turns out to be an easy consequence of the "successive minima" theorem from Minkowski's geometry of numbers [2] in the number field case and of Mahler's non-archimedean transcription of Minkowski [11] in the function field case. We use as well, of course, Riemann-Roch theorems in an arithmetic and a geometric context. Special attention is called to Remarks 2 and 3 of Section 3 in which we discuss the case of the relative dualizing sheaf $\omega_{X/S}$.

In the final section, we indicate the line of proof for higher dimensions.

I am grateful to Serge Lang and Barry Mazur for many discussions on Arakelov theory. My knowledge of the higher dimensional theory is entirely due to Christophe Soulé, and it was from his ideas that I developed this paper.

The author was partially supported by a Yale University Fellowship during the period of research.

1. Let X be an arithmetic surface over the ring of integers R in a number field F; i.e. X is a regular scheme, proper and flat over $S = Spec(R)$ of relative dimension one. We further assume that the (smooth) generic fiber X_F is geometrically irreducible of genus $g \geqslant 1$.

For each archimedean place v of F, choose an embedding $\sigma\colon F \to \mathbb{C}$ associated to v and denote by X_v the analytic space associated to the complex points of $X_F \times_F \mathbf{C}_\sigma$. As in Faltings [4], we can choose a basis $\{\omega_1, \omega_2, \ldots, \omega_g\}$ of holomorphic differentials on X_v orthonormal with respect to the inner product

Received November 3, 1989. Revision received March 5, 1990.