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HOROCYCLE FLOW ON GEOMETRICALLY FINITE SURFACES

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Let $S = \Gamma \backslash D^2$ be a quotient of the Poincaré disc by a finitely generated discrete group Γ of orientation preserving isometries acting without fixed points on D^2. Topologically S can be obtained from a compact surface by removing a finite number of closed discs.

The group of orientation preserving isometries of D^2 is $PSL(2, \mathbb{R})$ and the unit tangent bundle $T_1 S$ of S is a homogeneous space of $PSL(2, \mathbb{R})$:

$$T_1 S = \Gamma \backslash PSL(2, \mathbb{R}).$$

In particular, the unipotent subgroup of $PSL(2, \mathbb{R})$

$$N = \left\{ n(x) = \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix} : x \in \mathbb{R} \right\}$$

acts on $T_1 S$.

It is our main goal to determine all N-invariant Radon measures on $T_1 S$. Our first remark is that if C is the cone of positive N-invariant Radon measures in the space $\mathcal{M}(T_1 S)$ of all Radon measures with the vague topology, then C is the closed convex hull of the union of its extremal generators [B, II No. 2]; moreover it is easily seen that a measure is on an extremal generator of C if and only if it is ergodic. This reduces the problem to the classification of all ergodic measures.

To proceed further we consider the following decomposition of $T_1 S$: Let S^1 be the ideal boundary of D^2 and $\Lambda \subset S^1$ be the limit set of Γ. Using the visual map:

$$\text{Vis}: T_1 D^2 \to S^1,$$

we obtain first a decomposition of $T_1 D^2$ as a union of two subsets

$$\mathscr{F}_c = \{p \in T_1 D^2 : \text{Vis}(p) \in \Lambda\}$$

$$\mathscr{F}_d = \{p \in T_1 D^2 : \text{Vis}(p) \in S^1 \backslash \Lambda\}.$$

This gives via the projection $T_1 D^2 \to T_1 S$ a decomposition of $T_1 S$ into two disjoint subsets

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