Vol. 61, No. 3 DUKE MATHEMATICAL JOURNAL © December 1990

CUTTING AND PASTING AND THE η-INVARIANT

ROBERT MEYERHOFF AND DANIEL RUBERMAN

1. Introduction. This paper is a continuation of our earlier work [12], in which we studied the change in the η-invariant [1] of Riemannian 3-manifolds under the cutting and pasting operation of *mutation*. A mutation is the result of cutting a 3-manifold M along a genus 2 surface F, and regluing via the (unique) involution in the center of the mapping class group of F. When the mutation can be done geometrically, in particular for hyperbolic manifolds, we gave a formula for how the η-invariant of the new Riemannian manifold differs from that of the original manifold.

In the current paper, we substantially extend the results of [12] to calculate the change in the η-invariant of Riemannian 3-manifolds under geometric cutting and pasting along an arbitrary surface. The statement of the result, Theorem 1.3, requires some notation and definitions, but the general form of the theorem may be easily summarized. The change in the η-invariant is determined by topological invariants, involving the action of the gluing map on the surface along which the cutting and pasting occurs, and the embedding of the surface in the 3-manifold.

The proof of Theorem 1.3 is essentially topological. In particular, we do not need to make use of the "torsion formula" of Yoshida [18] in making our calculation. This provides a simplification of many of the arguments of [12] and gives an alternate approach to the theorems of that paper concerning the η-invariant. In addition, Theorem 1.3, and its proof, would apply with very few changes to $(4n - 1)$-dimensional Riemannian manifolds, cut and reglued along a codimension-one submanifold.

For the rest of the paper, all manifolds will be oriented. In particular, a codimension-one submanifold $N^n \subset V^{n+1}$ has a preferred trivialization of its normal bundle as $N \times [0, 1]$, with the orientation of V given by the orientation of N followed by $\partial/\partial t$. Before stating Theorem 1.3, we define the terms used in its statement and explain what we mean by geometric cutting and pasting.

Definition 1.1. Let V be a Riemannian manifold, and let N be a codimension-one submanifold smoothly embedded in V, with the metric induced from M. Suppose that $\varphi\colon N \to N$ is an isometry of N which extends to an isometry of a neighborhood $N \times (0, 1)$ of N in V. Then the cut and pasted manifold is defined by

$$V^{\varphi} =_{def} \left(V - N \times \frac{1}{2}\right) \cup N \times (0, 1)$$

Received February 22, 1990. Meyerhoff partially supported by NSF grant DMS 8602308.
Ruberman supported by an NSF Postdoctoral Research Fellowship.