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THE UNITARY DUAL OF THE UNIVERSAL COVERING GROUP OF $GL(n, \mathbb{R})$

JING-SONG HUANG

1. Introduction. In this paper we classify all the equivalence classes of unitary irreducible representations of the universal covering group of the general linear group over the real number field $\mathbb{R}$.

When n is larger than or equal to 3, let $\widetilde{GL}(n, \mathbb{R})$ denote the universal covering group of $GL(n, \mathbb{R})$; that is, the simply connected Lie group with maximal compact subgroup $\widetilde{K}(n)$, which is the universal covering group $Pin(n)$ of the orthogonal group $O(n)$. $\widetilde{GL}(n, \mathbb{R})$ is a disconnected and nonlinear reductive group. The problem of classifying the equivalence classes of unitary irreducible representations of a Lie group is well known as the unitary dual problem.

In [P], L. Pukanszky determined the unitary dual and wrote down the Plancherel formula for the universal covering group of $SL(2, \mathbb{R})$. In [Si], Dj. Sijacki determined the unitary dual of the universal covering group of $SL(3, \mathbb{R})$. Understanding of the unitary dual of the universal covering group of $SL(n, \mathbb{R})$ is very close to understanding the unitary dual of the universal covering group of $GL(n, \mathbb{R})$. So one can say that the problem was solved for $n = 2$ or 3. Unfortunately neither of the methods can be applied to larger n. The classification of unitary irreducible representations of the general linear group by D. Vogan suggests a way to solve the problem for any n.

$\widetilde{GL}(n, \mathbb{R})$ is just a double cover of $GL(n, \mathbb{R})$ for $n \geqslant 3$. Now we assume $\widetilde{GL}(1, \mathbb{R})$ and $\widetilde{GL}(2, \mathbb{R})$ are also the double covers of $GL(1, \mathbb{R})$ and $GL(2, \mathbb{R})$, respectively (not the universal covering groups). The idea of this convention is making the subgroup $\widetilde{GL}(m, \mathbb{R})$ inside $\widetilde{GL}(n, \mathbb{R})$ compatible with the subgroup $GL(m, \mathbb{R})$ inside $GL(n, \mathbb{R})$ through the projection map p. More precisely, suppose p: $\widetilde{GL}(n, \mathbb{R}) \to GL(n, \mathbb{R})$ is the projection, and $GL(m, \mathbb{R})$ is the subgroup of $GL(n, \mathbb{R})$; then $\widetilde{GL}(m, \mathbb{R})$ is the preimage of the projection p of $GL(m, \mathbb{R})$, i.e., the following diagram commutes:

$$\begin{array}{ccc} \widetilde{GL}(m, \mathbb{R}) & \hookrightarrow & \widetilde{GL}(n, \mathbb{R}) \\ \downarrow p & & \downarrow p \\ GL(m, \mathbb{R}) & \hookrightarrow & GL(n, \mathbb{R}). \end{array} \tag{1.1}$$

Since $\widetilde{GL}(n, \mathbb{R})$ is just a double cover of $GL(n, \mathbb{R})$, one can get about "half" of all

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