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HIGH FREQUENCY SCATTERING BY A CONVEX OBSTACLE

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1. Introduction and statement of results. The purpose of this paper is to provide a uniform analysis of the stationary wave diffracted by a convex obstacle in $\mathbf{R}^n$, both near the shadow boundary and near the boundary of the obstacle.

We characterize the region where the direct wave ("not hitting the obstacle") is equal to the free wave. The behaviour of the reflected wave is described in terms of double expansions depending on the distances to the obstacle and to the shadow boundary. The diffracted wave is expressed in terms of conormal distributions singular at "infinite frequency" and at the shadow boundary.

Investigation of the field scattered by obstacles has a long tradition in physics and applied mathematics.

The work of Fresnel on diffraction, which together with the remark of Poisson (the celebrated "Poisson spot") was the turning point for the wave theory of light, might be considered the beginning. Kirchhoff [8] raised Fresnel's work to the level of an elegant physical theory that remains inspiring to this day. Many others contributed to both qualitative and quantitative understanding of the subject. Of more recent date, one should mention among others the researches of Watson, Fock, Friedlander, Keller, Buslaev, Nussenzveig, Ludwig and Babich.

Following the work of Watson and Keller and Rubinow [9], Nussenzveig [19] carefully analyzed scattering by a sphere. He did not, however attempt to obtain uniform expansions across the transition layers which would have required either Watson's own refinement of Debye expansions of Bessel functions, or better, the uniform expansions of Olver (see Section 3 of [16]).

Ludwig [10, 11] proceeded from the spherical case to the case of an arbitrary convex obstacle using the analogy with his analysis of the behaviour at caustics and the expansions of Bessel functions. He also followed the ideas of Keller on the construction of the solutions of the eikonal equation. One cannot avoid the temptation of quoting the opening line of his paper [11] here[1]. His work was a starting point for Taylor's approach to propagation of singularities in the diffractive case [21].

Hörmander, in his work on Fourier integral operators [6], made geometrical optics a branch of pure mathematics, providing powerful tools and clarifying the relevant concepts. That enabled the analysis of scattering to become not only

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[1] "The asymptotic behavior of the field scattered by a convex object at high frequencies is extremely complicated."