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NONEQUIDIMENSIONAL VALUE DISTRIBUTION THEORY AND MEROMORPHIC CONNECTIONS

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This is a sequel to the paper [Si2] in which we introduced the use of meromorphic connections to handle nonequidimensional value distribution theory. The meromorphic connection is chosen so that the divisor has zero second fundamental form with respect to the meromorphic connection. When we have more than one divisor, they must have zero second fundamental form with respect to *the same* meromorphic connection. Such a condition is so stringent that it is difficult to find examples to which the method can be applied and which cannot be handled by other means. One example is a collection of Fermat curves in $\mathbb{P}_2$ of the form $a_0 w_0^d + a_1 w_1^d + a_2 w_2^d = 0$ for *the same* homogeneous coordinate system $[w_0, w_1, w_2]$. Shiffman [Sh2] told me that for such a collection of Fermat curves defect relations can be obtained by using a technique of H. Cartan [C2] (see §9), whereas the general case of allowing Fermat curves with respect to *different* homogeneous coordinates cannot be handled by any of the known methods. The only known result on the defect of general divisors is obtained by using the Veronese map and is given in [Sh1] in the following form. Let $f\colon \mathbb{C} \to \mathbb{P}_n$ be a holomorphic curve such that $f(\mathbb{C})$ is not contained in any algebraic hypersurface in $\mathbb{P}_n$. Let $\{S_j\}$ be a collection of algebraic hypersurfaces of degree d in $\mathbb{P}_n$. The sum of the defects of f for the collection of divisors $\{S_j\}$ is bounded from above by $\binom{n+d}{n}$ provided that the S_j's satisfy a suitable general position assumption. When applied to Fermat curves of degree d in $\mathbb{P}_2$ in different coordinate systems, the upper bound of the sum of the defects is of the order d^2. For Fermat curves of degree d in $\mathbb{P}_2$ in the same coordinate system, this is much greater than the known upper bound $6/d$ for the sum of the defects.

For the method of meromorphic connections introduced in [Si2], to consider *different* homogeneous coordinates necessitates the use of *different* meromorphic connections. One has to modify the method of [Si2] and consider what can be called a value distribution theory for meromorphic connections. In this paper we carry out such a modification. As a result of the modification done here, we can handle the case of divisors which have zero second fundamental forms with respect to meromorphic connections with the *same* pole order. In particular, one can handle the situation of a collection of Fermat curves with respect to *different* homogeneous coordinates. An upper bound of the sum of defects for Fermat curves of degree d in

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