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THE FUNDAMENTAL LEMMA FOR STABLE BASE CHANGE

LAURENT CLOZEL

1. Introduction. Let F be a p-adic field (local, non-Archimedean, of characteristic zero), $\mathcal{O}_F$ its ring of integers, ϖ_F a uniformizing parameter.

Let G be an unramified, connected reductive group over F. We will assume that G arises by base extension from a smooth reductive group scheme over $\mathcal{O}_F$; then $G(\mathcal{O}_F)$ is defined, as well as $H(\mathcal{O}_F)$ when H is a subgroup of G that can be obtained by base extension from a subgroup scheme defined over $\mathcal{O}_F$: in particular, this will apply to parabolic subgroups of G, their unipotent radicals or Levi components.

Let E/F be an unramified extension of degree l. We then have the base change map

$$b: \mathscr{H}_E \underset{l}{\rightarrow} \mathscr{H}_F$$

between the Hecke algebras of functions on $G(E)$, $G(F)$ invariant by $G(\mathcal{O}_E)$, $G(\mathcal{O}_F)$ (cf. e.g. [26]). We will write $K_L = G(\mathcal{O}_L)$, $L = E, F$. We denote by σ a generator of $\mathrm{Gal}(E/F)$ (for example the Frobenius element) and its action on F-subgroups of $G(E)$. Recall [25] that there is a *norm map* $\mathcal{N}$ sending (stable twisted conjugacy classes of) elements in $G(E)$ to stable conjugacy classes of elements in $G(F)$. For a semisimple element $\gamma \in G(F)$, its *stable orbital integral* is defined [26]. If γ is strongly regular, it is just

$$\Phi_f^{st}(\gamma) = \sum_{\gamma'} \Phi_f(\gamma'); \tag{1.1}$$

$f \in C_c^\infty(G(F))$, and the sum runs over a set of representatives for the conjugacy classes within the stable conjugacy class of γ. Analogously, we define

$$\Phi_{\varphi,\sigma}^{st}(\delta) = \sum_{\delta'} \Phi_{\varphi,\sigma}(\delta') \tag{1.2}$$

for $\delta \in G(E)$ such that $\mathcal{N}\delta$ is regular: $\varphi \in C_c^\infty(G(E))$, $\Phi_{\varphi,\sigma}(\delta')$ denotes the twisted orbital integral of φ at δ', and the sum runs over representatives for the σ-conjugacy classes within the stable σ-conjugacy class of δ.

The twisted centralizers, and the centralizers, that occur in the definition of the (twisted) orbital integrals, are all canonically isomorphic: one uses this isomorphism to define compatible Haar measures on them. On $G(E)$, $G(F)$ one takes the Haar

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