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ASYMPTOTIC BEHAVIOR OF FALTINGS'S DELTA FUNCTION

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Introduction. The purpose of this paper is to study Faltings's delta function, an invariant defined for any compact Riemann surface of positive genus. The delta function was originally defined by Faltings in [Fa] as part of the analytic theory in his study of arithmetic surfaces. Specifically, it was asked to determine the asymptotic behavior of the delta function near the boundary of the moduli space of stable curves of a fixed genus. We answer this question in Theorem 6.2 where we express Faltings's delta function as a generalized Weil function on the stable compactification of the moduli space of compact Riemann surfaces of a fixed genus. This fact is the main result of the paper.

This problem has been considered by others, most notably by Bost (unpublished) and Wentworth ([W]). In all cases the basic problem is the understanding of the canonical Green's function under degeneration. Our work differs from others in that we are using a different representation of the Green's function in terms of theta functions.

Briefly, the material is organized as follows. In Section 1 we present definitions and basic properties of Arakelov metrics, Green's function and Faltings's delta function. In Section 2 we use classical theta functions to represent all quantities in hand. In Sections 3 and 4 we study the degenerate behavior of these expressions as we consider a family of Riemann surfaces approaching a stable surface with nodes. In Section 5 we show how our results contain those of Faltings where he considered the case of elliptic curves. In Section 6 we summarize the degenerate behavior of the delta function by viewing it as a generalized Weil function. This type of expression was suggested by Lang in [La3].

Throughout our analysis of the degenerate behavior of the delta function, we only consider the simplest form of degeneration, namely, where the singular curve has one node. In particular, it is evident from our method that if one were to degenerate so that the limit curve has more than one node, the asymptotic behavior of the delta function would depend on the approach in the moduli space.

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1. Definitions and basic properties. Throughout $\mathbf{X}$ will denote a compact Riemann surface of genus $g \geqslant 1$. The canonical metric on $\mathbf{X}$ is defined as follows.

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