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AN APPROACH TO THE ABUNDANCE CONJECTURE FOR 3-FOLDS

KENJI MATSUKI

This paper presents an approach to the so-called Abundance Conjecture for 3-folds in the realm of Minimal Model Theory in dimension 3 recently established by S. Mori, Y. Kawamata and others. We prove

MAIN THEOREM. *Let X be a projective minimal Gorenstein 3-fold, i.e., a projective 3-fold having only terminal singularities of index one with K_X being nef. Assume some multiple of K_X has a member at least one of whose components is* <u>*not*</u> *birationally equivalent to a ruled surface. Then the Abundance Conjecture holds for X, that is to say, some high multiple of K_X is generated by global sections.*

The proof uses the results of Miyaoka [My1, 2, 3, 4] and Kawamata [Ka2, 4, 5] [KMM] and the thesis of the author [Mk2]. The theorem not only gives us a slight generalization of a theorem of P. M. H. Wilson in [W1], but also reduces the study of the conjecture to the one concerning the configuration of the ruled surfaces appearing in a member of the pluricanonical system.

As an easy corollary to this theorem, we have

COROLLARY. *Hyperbolicity* (*in the sense of Kobayashi*) *implies the Abundance Conjecture in dimension* 3.

From this corollary, it follows as in the argument of [Pe] with the use of Mori's theory that the conjecture of Kobayashi-Lang (cf. [Kb] [La]), claiming that a projective hyperbolic nonsingular variety X should have an ample canonical class K_X, can be reduced in dimension 3 to showing the following conjecture about Calabi-Yau manifolds (see also [W4] [Fr].):

CONJECTURE. *On a Calabi-Yau 3-manifold, there should exist a* (*possibly singular*) *rational or elliptic curve.*

For more elaborate efforts to prove the full Abundance Conjecture for 3-folds, we refer the reader to [My4] for the case $\nu = 1$ and [Mk2] for the case $\nu = 2$ using the notion of flops.

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