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MICROLOCAL PSEUDOCONVEXITY AND "EDGE OF THE WEDGE" THEOREM

PIERRE SCHAPIRA AND JEAN-MARIE TREPREAU

0. Introduction. Let X be a complex manifold, N a real analytic submanifold. First assume that N is the boundary of a pseudoconvex open set Ω of X, and that N contains no germ of complex curve. As an easy application of a result of Diederich and Fornaess [D-F3] we first prove that the sheaf of boundary values on N of holomorphic functions on Ω is flabby, and when $X = \mathbb{C}^n$ and Ω is bounded, we deduce a global decomposition theorem (cf. Corollary 2.2). Next consider the general case where N has codimension d and assume for simplicity that N is generic. Let U be an open subset of the conormal bundle T_N^*X such that the Levi form of N has exactly r negative eigenvalues on U. It is known by [K-S1] that the complex $\mu_N(\mathcal{O}_X)$, the Sato's microlocalization of $\mathcal{O}_X$ along N, is concentrated in degree $d + r$ on U. Making the extra assumption that U contains no germ of complex curve, we prove here that the sheaf $H^{d+r}(\mu_N(\mathcal{O}_X))$ is conically flabby on U, by reducing to the case where $d = 1$, $r = 0$, by a complex contact transformation. As an application, we obtain an "edge of the wedge" type theorem.

1. Pseudoconvex neighborhoods. In this section we construct special neighborhoods of pseudoconvex domains, which we shall use in the sequel. Theorem 1.2 below is sufficient for our applications, but we believe that Theorem 1.1 has its own interest.

THEOREM 1.1. *Assume Ω is a bounded pseudoconvex domain in $\mathbb{C}^n$ with real-analytic boundary $\partial\Omega$. Then for any open set ω of $\partial\Omega$, $\Omega \cup \omega$ has a fundamental system of pseudoconvex neighborhoods.*

Skipping Step 3 and Step 4 in the proof below of Theorem 1.1, one gets the following local statement, where Ω is an open subset of $\mathbb{C}^n$.

THEOREM 1.2. *Assume Ω is near $\zeta_0 \in \partial\Omega$ a pseudoconvex domain with real-analytic boundary $\partial\Omega$ such that $\partial\Omega$ contains no germ of complex curve. Then there exists an open neighborhood U of ζ_0 such that, for any open set ω in $\partial\Omega \cap U$, $(\Omega \cap U) \cup \omega$ has a fundamental system of pseudoconvex neighborhoods.*

Proof of Theorem 1.1.

Step 1. Let $\zeta_0 \in \partial\Omega$. It follows from Theorem 4 in [D-F2] that $\partial\Omega$ contains no germ of complex curve. Then, a weakening of Theorem 2 in [D-F3] gives the existence of an open neighborhood U of ζ_0 and of a continuous function

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