Vol. 60, No. 3 DUKE MATHEMATICAL JOURNAL © June 1990

GLOBAL EXISTENCE OF SMALL ANALYTIC SOLUTIONS TO NONLINEAR SCHRÖDINGER EQUATIONS

NAKAO HAYASHI

1. Introduction. In this paper we consider the following nonlinear Schrödinger equation in $\mathbb{R}^n$ ($n \geqq 2$):

$$i\partial_t u + \tfrac{1}{2}\Delta u = F(u, \nabla u, \bar{u}, \overline{\nabla u}), \quad (t, x) \in \mathbb{R} \times \mathbb{R}^n, \tag{1.1}$$

$$u(0, x) = \phi(x), \qquad x \in \mathbb{R}^n. \tag{1.2}$$

Here the nonlinear term $F: \mathbb{C} \times \mathbb{C}^n \times \mathbb{C} \times \mathbb{C}^n \to \mathbb{C}$ is a polynomial of degree 3 satisfying

$$|F(u, \nabla u, \bar{u}, \overline{\nabla u})| \le C \cdot (|u| + |\nabla u|)^3$$

and

$$F(\omega u, \omega \nabla u, \overline{\omega u}, \overline{\omega \nabla u}) = \omega F(u, \nabla u, \bar{u}, \overline{\nabla u}),$$

for any complex number ω with $|\omega| = 1$, and ∇ stands for the nabla with respect to x.

Our main purpose in this paper is to discuss the global existence and analyticity of small solutions of (1.1)–(1.2) under a certain analytical condition on ϕ. The proof presented here is based on a modification of the method used in the previous paper [2] in which we only consider the special case $F = |u|^2 u$. It seems that the method of [2] does not work for (1.1)–(1.2) directly.

We state notations and function spaces used in this paper. In particular we introduce new function spaces which help to make a proof more simple than the previous one [2].

Notation and function spaces. We let $L^p(\mathbb{R}^n) = \{f(x); f(x)$ is measurable on $\mathbb{R}^n$, $\|f\|_{L^p} < \infty\}$ where $\|f\|_{L^p} = (\int_{\mathbb{R}^n} |f(x)|^p\, dx)^{1/p}$ if $1 \leqq p < \infty$ and $\|f\|_{L^\infty} = \text{ess.sup}\{|f(x)|; x \in \mathbb{R}^n\}$ if $p = \infty$, and we let $H^{m,p}(\mathbb{R}^n) = \{f(x) \in L^p(\mathbb{R}^n); \|f\|_{H^{m,p}} = \sum_{|\alpha| \leqq m} \|\partial_x^\alpha f\|_{L^p} < \infty\}$, where $\alpha = (\alpha_1, \ldots, \alpha_n) \in (\mathbb{N} \cup \{0\})^n$ is a multi-index, $\partial_x^\alpha = \partial_{x_1}^{\alpha_1} \ldots \partial_{x_n}^{\alpha_n}$ and $|\alpha| = \alpha_1 + \cdots + \alpha_n$. We denote by $\wedge$ and $\mathscr{F}^{-1}$ the Fourier transform and inverse, respectively. For each $r > 0$ we denote by $S(r)$ the strip $\{-r < \operatorname{Im} z_j < r; 1 \leqq j \leqq n\}$ in $\mathbb{C}^n$. For $x \in \mathbb{R}^n$, if a complex-valued function $f(x)$ has an analytic continuation to $S(r)$, then we denote this by the same letter $f(z)$ and if $g(z)$ is an analytic function on $S(r)$, then we denote the restriction of $g(z)$ to the real axis by $g(x)$. We let

$$AL^p_\infty(r) = \{f(z); f(z) \text{ is analytic on } S(r), \|f\|_{AL^p_\infty(r)} < \infty\},$$

Received November 6, 1989.