Vol. 60, No. 3 DUKE MATHEMATICAL JOURNAL © June 1990

ON AMPLE VECTOR BUNDLES WHOSE ADJUNCTION BUNDLES ARE NOT NUMERICALLY EFFECTIVE

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1. Introduction. By a generalized polarized variety we mean an algebraic variety X together with an ample vector bundle E on it, i.e., a pair (X, E). In particular when E is a line bundle, this is nothing but the usual notion of polarized variety. We also define $K_X + c_1(E)$ to be its associated adjunction bundles. In this paper we are going to study the numerical effectiveness of this bundle. Inspired by some conjectures of Mukai in [16] we generalize most of the results in Fujita's paper [3] about adjunction bundles of polarized varieties to those of generalized polarized varieties. As a result, we are able to solve some conjectures of Mukai and a problem of Sommese.

Here is an outline of this paper. In section two we fix some notations and conventions. In section three we state our main theorems. In the final section we give the proofs of our main results.

The authors would like to express their sincere thanks to their advisor Professor David Morrison, for his guidance and encouragement. We also thank Professor Lawrence Ein for pointing out to us a result concerning vector bundles on hyper-quadrics. We are also grateful to the referee for clarifying some ambiguities in the first draft of this paper.

2. Notations and Conventions. Throughout this paper we restrict ourselves to the category of schemes over an algebraically closed field k of characteristic zero. Most of our notations are common ones, but for the purpose of completeness we are now giving a brief list of our notations as follows:

Let X and Y be two k-schemes.

- $T_{X/Y}$: the relative tangent bundle if X is smooth over Y. When Y is a point, we simply write it as T_X.
- $K_{X/Y}$: the relative canonical bundle. As above when Y is a point, we denote it by K_X.
- $H^i(X, \mathscr{F})$: the i^{th} cohomology group for a coherent sheaf $\mathscr{F}$.
- $h^i(X, \mathscr{F})$: the dimension of the cohomology group, i.e., $\dim_k H^i(X, \mathscr{F})$.
- $c_i(E)$: the i^{th} Chern class of E.
- $P(E)$: the projectivized bundle associated with E, or $\mathrm{Proj}(\bigoplus^m S^m(E))$.
- $\chi(X, E) := \Sigma_{i=0}^n (-1)^i h^i(X, E)$, sometimes simply $\chi(E)$.
- $\rho(X)$: the Picard number of X, i.e. the rank of $\mathrm{Pic}(X)$.

Received July 6, 1989. Revision received October 23, 1989.