Vol. 60, No. 3 DUKE MATHEMATICAL JOURNAL © June 1990

HÖLDER REGULARITY OF SUBELLIPTIC PSEUDODIFFERENTIAL OPERATORS

PENGFEI GUAN

Introduction. Since Hans Lewy's celebrated example of a nonsolvable operator, much important work has been done on the study of solvability and hypoellipticity of pseudodifferential operators. (e.g., [H1], [NT1], [NT2], [T], [BF], [E1], [H3], etc.) The importance of study of the pseudodifferential operators is also illustrated by its connection to other problems ([H2], [FK]).

The following condition (Ψ) and condition (P) introduced by Nirenberg and Treves are fundamental.

Condition (Ψ):

Let $p(x, \xi)$ be the principal symbol of P, given any point $(x_0, \xi^0) \in \Omega \times \{\mathbb{R}^n \backslash \{O\}\}$, and such that there exists some $z \in C\backslash\{O\}$,

$$p(x_0, \xi^0) = 0, \qquad d_\xi \operatorname{Re}(zp)(x_0, \xi^0) \neq 0$$

the function $\operatorname{Im}(zp)(x, \xi)$, restricted to the bicharacteristics curve of $\operatorname{Re}(zp)(x, \xi)$ through (x_0, ξ^0) can never change sign from $-$ to $+$ when one moves in the positive direction.

Condition (P):

The function $\operatorname{Im}(zp)(x, \xi)$ does not change sign in the bicharacteristic curve of $e(zp)(x, \xi)$ through (x_0, ξ^0).

Condition (P) is necessary and sufficient for local solvability of differential operators. ([NT2], [BF]). Condition (Ψ) is necessary for local solvability of pseudodifferential operator P and for hypoellipticity of $\bar{P}$. ([M]). For the subellipticity, a very strong result of Egorov characterizes all subelliptic operators. A pseudodifferential operator P is subelliptic if and only if $\bar{P}$ satisfies condition (Ψ) and the function $\operatorname{Im}(zp)$ vanishes to only finite order along the bicharacteristic curves of $\operatorname{Re}(zp)$. If k is the smallest number such that the order of vanishing is less than or equal to k, then we can take $\delta = 1/(k + 1)$; this is sharp. Egorov's original proof ([E1]) of the sufficiency does not seem to be very rigorous. The first complete proof for differential operators was given by Treves ([T]). Hörmander ([H3]) later supplied the missing part in the proof of [E1]. (see also [F]).

In this paper, we study Hölder regularity of subelliptic pseudodifferential operators. It is well known that if P is an elliptic pseudodifferential operator of order m, then

$$u \in D'(\Omega), \qquad Pu \in \Lambda^{\alpha}_{\text{comp}}(\Omega) \rightarrow u \in \Lambda^{\alpha+m}_{\text{loc}}(\Omega),$$

Received August 7, 1989. Revision received October 11, 1989.