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CORRECTION TO ON ISOSPECTRAL LOCALLY SYMMETRIC SPACES AND A THEOREM OF VON NEUMANN

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I am very grateful to A. Reid for alerting me to an error in [S]. Lemma 4.1 in [S] is false in general. This invalidates the proof that the lattices Γ_1 and Γ_2 constructed there are not isomorphic. Here we will show that these two lattices indeed are not isomorphic. For our notations we refer to [S].

Suppose to the contrary that $\phi: \Gamma_1 \to \Gamma_2$ is an isomorphism. Since $\mathbf{G}$ is simple and not locally isomorphic to $SL(2, \mathbf{R})$, ϕ extends to an automorphism of $\mathbf{G}$ by Mostow's rigidity theorem. Let Φ be the induced automorphism of the Lie algebra of $\mathbf{G}$. Since $\operatorname{Ad} \phi(g) = \Phi \operatorname{Ad}(g)\Phi^{-1}$ we see that $\operatorname{tr} \operatorname{Ad} \phi(g) = \operatorname{tr} \operatorname{Ad} g$. In the construction of Γ_1 and Γ_2 we may and will suppose that $p > n$ where n is the dimension of $\mathbf{G}$.

In the Heisenberg group H_2 set

$$a = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad b = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{and} \qquad c = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Then $c = [a, b]$. Pick lifts $\tilde{a}$ and $\tilde{b}$ of a and b in Γ_2 and set $\tilde{c} = [\tilde{a}, \tilde{b}]$. Then $\tilde{c}$ is a lift of c to Γ_2. As H_1 is abelian we see that $\phi^{-1}(\tilde{c}) \in [\Gamma_1, \Gamma_1] \subset C$. Hence $\operatorname{tr} \operatorname{Ad}(\tilde{c}) = \operatorname{tr} \operatorname{Ad} \phi^{-1}(\tilde{c}) \equiv n \bmod p$. On the other hand, recall from the proof of Lemma 3.2 that $\mathscr{A}_{27}$ and thus H_2 are embedded into $\mathbf{G}'_{\mathrm{ad}}(\mathbf{F}_p)$ by permuting coordinates. It follows that $\operatorname{Ad}(c)$ permutes the root spaces and is hence represented by a permutation matrix of 0's and 1's (with respect to a Chevalley basis of the Lie algebra). Since $\operatorname{Ad}(c) \neq 1$ and $p > n$, it follows that $\operatorname{tr} \operatorname{Ad}(c) \not\equiv n \bmod p$. Since $\operatorname{tr} \operatorname{Ad}(c) \equiv \operatorname{tr} \operatorname{Ad}(\tilde{c}) \equiv n \bmod p$, this is a contradiction.

References

[S] R. J. Spatzier, On isospectral locally symmetric spaces and a theorem of von Neumann, Duke Mathematical J. **59** (1989), 289–294.

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