Vol. 60, No. 2 DUKE MATHEMATICAL JOURNAL © April 1990

HODGE THEORY ON HYPERBOLIC MANIFOLDS

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Dedicated to David Gilbarg on the occasion of his seventieth birthday.

1. Introduction. The classical Hodge theorem is a simple yet remarkable link between analysis and topology on an arbitrary compact Riemannian manifold. It has no analog on a general noncompact manifold, even if the manifold is assumed complete, for the simple reason that spectral properties of the Laplacian are very sensitive to the behavior of the metric at infinity. Still, it is to be hoped that some Hodge-type theorem remains true when the manifold is metrically simple. For example, it is natural to ask whether there is a Hodge theorem on complete spaces of constant curvature. We show in this paper that this is the case for geometrically finite complete manifolds of constant negative curvature. What is new is our treatment of manifolds with cusps as well as forms of middle degree $(n-1)/2 \leqslant k \leqslant (n+1)/2$. It turns out that cusps play a complicated and unexpected role.

If M is a Riemannian manifold then the exterior derivative d, which maps (for example) smooth compactly supported differential forms of degree k to forms of degree $k+1$, $0 \leqslant k \leqslant n = \dim M$

$$d\colon C_0^\infty\Omega^k(M) \to C_0^\infty\Omega^{k+1}(M),$$

has an adjoint δ such that

$$\delta\colon C_0^\infty\Omega^k(M) \to C_0^\infty\Omega^{k-1}(M).$$

The Hodge Laplace operator

$$\Delta = d\delta + \delta d\colon C_0^\infty\Omega^k(M) \to C_0^\infty\Omega^k(M)$$

is a second order symmetric elliptic differential operator. It is nonnegative by virtue of the integration by parts

$$\langle\Delta\omega, \omega\rangle = \|dw\|^2 + \|\delta\omega\|^2 \geqslant 0, \qquad \omega \in C_0^\infty\Omega^k. \tag{1.1}$$

When the metric on M is assumed complete, an old theorem due to Gaffney [G1]

Received June 12, 1989. Revision received October 2, 1989.
Research supported in part by the NSF under a Postdoctoral Research Fellowship and Grant #DMS-8903076.