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ON LONG-RANGE SCATTERING

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0. Introduction. Even if special cases are not well understood it pays sometimes to consider a problem at hand in the most general framework. Thus addressing the existence of wave operators for the most general potentials Hörmander [Hö1] has discovered a natural asymptotic evolution for the long-range scattering. It is determined by a solution, $S(k, t)$, of a certain Hamilton-Jacobi equation. This discovery was immediately used in the investigation of asymptotic completeness for one-particle long-range scattering [Com, Hö2, IK, K1, 2, KY, M, S1,2]. (References on the other results in the long-range scattering can be found in [En, Hö2, RS III].) Functions closely related to S were used to construct approximations to the full time evolutions and to find the asymptotic behaviour of the generalized eigenfunctions of the continuous spectrum. We take a different route. We use S to find the correct time asymptotics for the coordinate x. Namely, we show that

$$\left\|\left(x - \frac{\partial S}{\partial p}\right)\Psi_t\right\| \leqslant \text{const}$$

on Schrödinger orbits $\Psi_t = e^{-iHt}\Psi$ for a dense set of Ψ's (Theorem 5.1, cf [Pe]). Here $p = -i\,\text{grad}_x$, $\partial S/\partial p = (\text{grad}_k\, S)(p, t)$ and H is a Schrödinger operator in question. This sharp propagation estimate yields readily asymptotic completeness for rather general long-range systems (theorem 7.1). We extend these results to a certain class of time dependent Hamiltonians, introduced in [SigSof 1] (theorem 8.1). Note in passing that Hamiltonians from the latter class arise naturally in the investigation of the many-body long-range problem [SigSof 2]. It would be interesting to extend the outlined method to Schrödinger operators of the form

$$H(x, p) = \omega(x, p) + V(x, p)$$

with $\omega(x, k)$, a smooth symbol homogeneous degree 0 in x for $|x| \geqslant 1$ and obeying (1)–(3) of section 1 and with $V(x, k)$ obeying (1.4).

A word about notation. Below we will deal with n-tuples of operators $A = (A_1, A_2, \ldots, A_n)$. The $\mathbb{R}^n$-norm of A will be denoted by $|A|$:

$$|A| = (\Sigma\, |A_i|^2)^{1/2},$$

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