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LONG TIME EXISTENCE OF A CLASS OF PERTURBATIONS OF PLANAR SHOCK FRONTS FOR SECOND ORDER HYPERBOLIC CONSERVATION LAWS

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1. Introduction. In this paper we consider shock front solutions of a second order hyperbolic conservation law of the form

$$\sum_{0 \leqslant i \leqslant N} \partial_i(H^i(\varphi')) = 0, \tag{1.1}$$

where $\varphi' = (\partial_0\varphi, \partial_x\varphi)$, $\partial_i = \partial/\partial x_i$, $\partial_x\varphi = (\partial_1\varphi, \ldots, \partial_N\varphi)$. Here $x_0 > 0$ is the time variable (sometimes also denoted by t), $x = (x_1, \ldots, x_N) \in \mathbb{R}^N$, and $H^i \in C^\infty(D, \mathbb{R})$, where D is an open subset of $\mathbb{R}^{N+1}$. By a shock front solution of (1.1) in an open neighbourhood Ω of $(\bar{x}_0, \bar{x}) \in \mathbb{R}^+ \times \mathbb{R}^N$, we mean that there exist a C^1 hypersurface S of Ω, transversal to the planes $x_0 =$ constant, dividing $\Omega\backslash S$ into two parts Ω^- and Ω^+, and a weak solution $\varphi \in C(\bar{\Omega})$ of (1.1) such that

(i) $\varphi^\pm = \varphi|_{\Omega^\pm} \in C^2(\overline{\Omega^\pm})$ and $\varphi^\pm$ is a classical solution of (1.1);
(ii) S is noncharacteristic for $\sum_{0 \leqslant i, j \leqslant N} \frac{1}{2}(\partial_j H^i + \partial_i H^j)((\varphi^\pm)')\partial_{ij}^2$.

In [10], [8], Majda and Thomann assumed a Lax type entropy condition, a multi-D stability condition, and compatibility conditions for initial datas. Then they obtained local existence of shock fronts with these initial data, via a reduction to a hyperbolic mixed problem by means of a partial hodograph transform. In the present paper, we start with a global planar shock front solution of (1.1), satisfying the conditions of [10], [8], and show that small C^∞ compatible perturbations with compact support of the data of φ at the timelike side of S (see below) give rise to shock fronts with long lifespan (actually global in time if $N \geqslant 5$) which remain stable and almost planar. We obtain our results by reducing to a mixed problem via the partial hodograph transform, following Majda-Thomann, and by studying this mixed problem with an adaptation of techniques developed for the Cauchy problem by Klainerman [5], [7].

Our paper is organized as follows. In Section 2 we describe our results precisely. In Section 3 we reduce the problem via the partial hodograph transform. The proofs of the results announced in Section 2 are then completed in Sections 4 and 5. We shall use some results of [9] about continuation properties of solutions of nonlinear hyperbolic mixed problems, and also some extensions of these results. In order not to interrupt the proofs in Section 4 and 5, we discuss the needed continuation

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