Vol. 60, No. 2 DUKE MATHEMATICAL JOURNAL © April 1990

HECKE POINTS ON MODULAR CURVES

JOSEPH H. SILVERMAN

Let Γ be one of the arithmetic groups $\Gamma_0(N)$ or $\Gamma_1(N)$ or $\Gamma(N)$, and let $X = X(\Gamma)$ be the corresponding modular curve. For every integer m relatively prime to N, the Hecke correspondence

$$T_m \subset X \times X$$

induces a map

$$T_m\colon X(\bar{\mathbb{Q}}) \to \mathrm{Div}_{\bar{\mathbb{Q}}}(X).$$

If $x \in X(\bar{\mathbb{Q}})$ and

$$T_m(x) = \sum_{i=1}^{\sigma_1(m)} (y_i) \in \mathrm{Div}_{\bar{\mathbb{Q}}}(X),$$

we will call the y_i's the **m-Hecke points associated to x**. Here $\sigma_1(m)$, the sum of the divisors of m, is the degree of $T_m(x)$. In this paper we will describe the Diophantine behavior of these Hecke points, both as a function of x and as a function of m. (For basic facts about modular curves and Hecke correspondences, see [10] or [11]. Our results are in fact valid for modular curves $X(\Gamma)$ for any of the arithmetic groups Γ described by [11, equation 3.3.2, page 67].)

An important measure of the Diophantine complexity of a point is its height. Let $\infty \in X$ be the cusp "at infinity," and choose an (absolute logarithmic) Weil height function

$$h_X\colon X(\bar{\mathbb{Q}}) \to [0, \infty)$$

on X corresponding to the divisor (∞). (For basic facts about height functions, see [6].) Using a natural notation, we define

$$h_X(T_m(x)) = \sum_{i=1}^{\sigma_1(m)} h_X(y_i), \quad \text{if} \quad T_m(x) = \sum_{i=1}^{\sigma_1(m)} (y_i).$$

Our first result shows that for fixed m and varying x, the m-Hecke points associated to x have, on average, about the same height as x.

Received February 2, 1989. Revision received July 27, 1989. Research partially supported by NSF DMS-8842154.