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WEIGHTED ESTIMATES FOR SINGULAR INTEGRALS VIA FOURIER TRANSFORM ESTIMATES

DAVID K. WATSON

In [KW1], D. Kurtz and R. Wheeden proved weighted estimates for homogeneous singular integral operators on $\mathbf{R}^n$ which satisfy a "L^r-Dini condition." In this paper we show that the smoothness requirement of the L^r-Dini condition is in fact unnecessary. Moreover, we are able to broaden the family of singular integral operators substantially. This is done by a technique originally developed for Hilbert transforms along curves, using a Fourier transform estimate developed in [DR]. The proof continues the philosophy begun in [NRW] and continuing with refinements in [SWa], [NW], [CS], [C+], [DR], [C²VW²], [F], [N], and others that Fourier transform estimates may be used to substitute for smoothness requirements in singular integral estimates.

Throughout the paper, p' will denote the dual exponent to p, that is $1/p + 1/p' = 1$. We prove:

THEOREM 1. *Let $n \geqslant 2$, $1 < r < \infty$, and let $Tf(x) = p.v.\ f * k(x)$ for*

$$k(x) = h(|x|)\frac{\Omega(x)}{|x|^n}, \tag{1}$$

where Ω is homogeneous of degree 0 on $\mathbf{R}^n$, $\Omega \in L^r(\mathbf{S}^{n-1})$, Ω has average 0 on $\mathbf{S}^{n-1}$, and h is a measurable function on $(0, \infty)$ satisfying:

$$\int_R^{2R} |h(t)|^r\, dt \leqslant CR \qquad \text{for all } R > 0. \tag{2}$$

Then T is bounded on $L^p(w)$,

$$\|Tf\|_{p,w} \leqslant B_{p,w}\|f\|_{p,w}, \tag{3}$$

in each of the following situations:

(A) *If $r' \leqslant p < \infty$, $p \neq 1$, and $w \in A_{p/r'}$, or*
(B) *If $1 < p \leqslant r$, $p \neq \infty$, and $w^{-1/(p-1)} \in A_{p'/r'}$, or*
(C) *If $1 < p < \infty$ and $w^{r'} \in A_p$, or*
(D) *If $1 < p < \infty$, and $w = |x|^\beta$ for $\mathrm{Max}\{-n, -np/r'\} < \beta < \mathrm{Min}\{n(p-1), np/r'\}$. In particular, when $r \geqslant 2$, then for $r' \leqslant p \leqslant r$, β lies in the maximum A_p range $-n < \beta < n(p-1)$.*

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