PROPER HOLOMORPHIC MAPS FROM WEAKLY PSEUDOCONVEX DOMAINS

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Introduction. Let D be a bounded pseudoconvex domain with real-analytic boundary in $\mathbb{C}^2$. In this paper we prove the existence of proper holomorphic maps from D into higher-dimensional polydiscs and balls. In fact, we prove the following:

THEOREM. *Suppose $D \subset\subset \mathbb{C}^2$ is a pseudoconvex domain with real-analytic boundary.*

(2.1)   *There exists a proper holomorphic map from D into the unit polydisc in $\mathbb{C}^3$.*

(4.1)   *There exists a uniformly continuous proper holomorphic map from D into the unit ball in $\mathbb{C}^3$.*

The existence of proper holomorphic maps from strongly pseudoconvex domains to higher-dimensional polydiscs was proved independently by Aleksandrov [AL2] and Løw [LO3, LO4]. Løw also constructed maps to balls, as did Forstnerič [FR], and these results have been refined in various ways by several authors. The class of weakly pseudoconvex domains has been studied much less in this regard (although Aleksandrov's paper does treat certain convex, but not strongly pseudoconvex, domains.) A notable counterexample is due to Sibony [SI]: There exists a smoothly bounded pseudoconvex Reinhardt domain in $\mathbb{C}^2$ which cannot be mapped properly into any bounded convex domain in complex Euclidean space. This answered a question raised in [HT, p. 64].

One obstacle to extending earlier constructions for strongly pseudoconvex domains was the dependence of the codimension of the constructed map on the domain; as the Levi form degenerated, the codimension apparently grew without bound. This dependence was avoided in the techniques introduced by Stensønes [ST1], who showed that each smoothly bounded strongly pseudoconvex domain in $\mathbb{C}^2$ can be mapped properly into the unit polydisc in $\mathbb{C}^3$; in [ST2] Stensønes gave a generalization to domains of higher dimension. The techniques of [ST1] were used by Dor [DO] and Hakim [HA] independently to give the optimal codimension of one for maps to balls. In [ST2, Theorem 3] it was proved that there exist proper holomorphic maps to polydiscs from pseudoconvex domains in $\mathbb{C}^n$ whose weakly pseudoconvex boundary points lie in a finite union of maximum modulus sets. There it was asked whether the conclusion is valid for real-analytic pseudoconvex domains; our main result is an affirmative answer for $n = 2$.