Vol. 60, No. 1 DUKE MATHEMATICAL JOURNAL © February 1990

THE BURNS-EPSTEIN INVARIANT AND DEFORMATION OF CR STRUCTURES

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1. Introduction. The theory of CR structures in dimension 3 differs strikingly from that in higher dimensions. One reason is that there is no integrability condition for 3-dimensional CR structures, and every compact orientable 3-manifold possesses many nondegenerate CR structures. Among all such structures, the ones that are geometrically most interesting are the *spherical* ones (in the terminology of [**BS**]): those that are locally CR equivalent to the unit sphere in $\mathbb{C}^2$. The question of which compact 3-manifolds admit spherical CR structures is of considerable geometric and topological interest, since any such manifold that is simply connected admits a CR diffeomorphism with the 3-sphere [**BS**].

In [**BE1**], D. Burns and C. Epstein defined a real-valued global invariant μ of certain CR structures on compact 3-dimensional manifolds, and showed that the critical points of μ, viewed as a functional on the space of CR structures, are exactly the spherical structures. Their construction required a global holomorphic vector field, so μ is defined only for CR structures whose holomorphic tangent bundle is trivial. In [**BE2**] they extended their construction to boundaries of strictly pseudoconvex domains in higher dimensions.

This paper has two purposes. First, we extend the definition of the Burns-Epstein invariant to arbitrary oriented compact 3-dimensional CR manifolds. To do so, we must reinterpret it as an invariant of a *pair* of CR structures (see Definition 2.5); it becomes an invariant of a single CR structure once a reference structure is chosen. In §2 we prove the following theorem.

THEOREM A. *Let M be an oriented compact 3-manifold with a given oriented contact structure H. For any oriented contact form θ for H, and any two oriented CR structures J and $\hat{J}$ compatible with H, let $\mu(J, \hat{J}) = \mu_\theta(J, \hat{J}) \in \mathbb{R}$, defined by formula* (2.12).

(i) *$\mu(J, \hat{J})$ depends only on the CR structures J and $\hat{J}$, not on the choice of contact form θ.*

(ii) *$\mu(J, \hat{J}) - \mu(J, \tilde{J}) = \mu(\tilde{J}, \hat{J})$, so $\mu(J, \hat{J})$ depends on the choice of $\hat{J}$ only up to an additive constant.*

(iii) *If the holomorphic tangent bundles of J and $\hat{J}$ are trivial, then $\mu(J, \hat{J}) = \mu(J) - \mu(\hat{J})$, where μ is the Burns-Epstein invariant.*

Received February 17, 1989. Revision received June 23, 1989. First author supported in part by National Science Council grant NSC 77-0208-M0001-20 (R.O.C.). Second author supported in part by National Science Foundation grant DMS-8702618 (U.S.A.).