Vol. 60, No. 1 DUKE MATHEMATICAL JOURNAL © February 1990

PROPAGATION OF GEVREY SINGULARITIES FOR A CLASS OF OPERATORS WITH TRIPLE CHARACTERISTICS, I.

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0. Introduction. It is well known that the Cauchy problem in the C^∞ category for hyperbolic operators with multiple characteristics is well-posed, only if in general some Levi conditions on the lower order terms are satisfied. For instance, in the case of double characteristics, complete results have been obtained by Ivrii-Petkov [7], Hörmander [3] and Ivrii [6]. On the other hand if one considers the same problem in the Gevrey classes G^s, one realizes that Levi conditions on the lower order terms have to be imposed only if the Gevrey index s is greater or equal to a certain critical index, which in this case is equal to 2. For a very complete description of propagation of G^s singularities in case of hyperbolic operators with double characteristics, see B. Lascar [9]. For general hyperbolic operators with multiple characteristics almost nothing is known about the conditions on the lower order terms which guarantee the well-posedness of the Cauchy problem in the C^∞ category. For general results when s is sufficiently small, however, see Kajitani-Wakabayashi [11]. In this paper our aim has been to prove a propagation result in G^s, $1 < s < +\infty$, in the form of a microlocal Holmgren uniqueness theorem, for a class of hyperbolic operators with triple characteristics. We now have three critical indices, $\frac{3}{2}$, 2, 3 and here it has been possible to elucidate what kind of Levi conditions are sufficient depending on where s is located, in order to get microlocal uniqueness. We also would like to remark that these conditions are very close to being necessary, see e.g. [1] where this has been proved for a different, but similar class of operators. Let us now introduce notations and state precisely our result: Let $x = (x_0, x') \in \mathbb{R}^{n+1}$, $x' = (x_1, \dots, x_n)$ and $(x, \xi) \in \dot{T}^*\mathbb{R}^{n+1}$, we shall consider in the following Gevrey symbols of order m, p, i.e., C^∞ functions such that: $\exists C, A > 0$:

$$|D_x^\alpha D_\xi^\beta p(x, \xi)| \leqslant CA^{|\alpha|+|\beta|}(\alpha!)^s(\beta!)^s\langle\xi\rangle^{m-|\beta|}$$

for $x, \xi \in \mathbb{R}^{n+1}$, $\langle\xi\rangle = (1 + |\xi|^2)^{1/2}$. Let then be P a pseudodifferential Gevrey s operator, whose Weyl symbol has the form:

$$p(x, \xi) = p_m(x, \xi) + p_{m-1}(x, \xi) + p_{m-2}(x, \xi) + q(x, \xi), \tag{1}$$

where q is a Gevrey s symbol of order $m - 3$ and p_{m-j} is positively homogeneous of order $m - j$. Let us now recall that $f \in G^s(\Omega)$, Ω open subset in $\mathbb{R}^{n+1}$, if $\forall K \subset\subset$

Received December 27, 1988. Revision received June 5, 1989.