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BOUNDARY BEHAVIOR OF RATIONAL PROPER MAPS

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1. Introduction. Assume Ω and Ω' are relatively compact in $\mathbb{C}^n$ and $\mathbb{C}^k$ respectively with $k \geqslant n$ and let f be a proper holomorphic mapping of Ω into Ω'. Several authors ([2], [3], [4], [6], [8], [9], and [14]) have shown that by assuming certain conditions on the boundaries of Ω and Ω' and by assuming smooth extensions of f to the boundary of Ω that f has a holomorphic extension to a domain containing the closure of Ω. There are other useful types of assumptions that allow one to continue proper holomorphic mappings f, such as the graph of f being contained in a particular subvariety of $\mathbb{C}^n \times \mathbb{C}^n$, [1].

In [9], F. Forstneric proved that if Ω and Ω' are balls in $\mathbb{C}^n$ and $\mathbb{C}^k$ respectively and if f is proper and smooth of class C^{k-n+1} then f is a rational mapping. He posed the question whether every proper, rational mapping of B_n into B_k is holomorphic on the closure of B_n. We prove that this is so as a corollary to our main theorem. It is noted on p. 33 of [9] that Pinchuk has an unpublished version of this result. The principal result of our paper is the following.

Suppose f is a bounded holomorphic mapping of B_n into $\mathbb{C}^k$ and that f is "locally a proper mapping," (in the sense that will be made clear in the formal statement of our theorem) near a boundary point b of B_n and $f = (1/q)P$ near b where P is a holomorphic mapping and q is a complex valued holomorphic function in a neighborhood of b. Then f can be extended to a holomorphic mapping on some neighborhood of b.

Not all proper mappings of B_n into B_k can be extended holomorphically over $\bar{B}_n$, (see [5], [7], [11], and [13]). We give an example of a domain $\Omega \subset \mathbb{C}^2$ and a proper rational mapping $f: B_2 \to \Omega$ that is not holomorphic on the closure of B.

2. Statement and Proof of Main Theorem. Our main theorem is as follows.

THEOREM. *Assume f is a holomorphic, bounded mapping of B_n into $\mathbb{C}^k$ and that $b \in \partial B_n$. Assume further that there is a neighborhood $N = \{\|z - b\| < \varepsilon\}$ of b, a vector valued holomorphic mapping $P: N \to \mathbb{C}^k$, and a complex valued holomorphic function $q: N \to \mathbb{C}$ such that $f(z) = 1/q(z)P(z)$ when $z \in N \cap B_n$. Finally, assume that $f(N \cap B_n) \subset B_k$ and that for every sequence $\{z^{(j)}\} \subset N \cap B_n$ with $\|z^{(j)}\| \to 1$, we have $\|f(z^{(j)})\| \to 1$. Then f extends to a holomorphic mapping on $\{\|z - b\| < \varepsilon'\} \cup B_n$ for some $\varepsilon' > 0$.*

Using a result of Forstneric [9, Theorem 2], we obtain the following corollary.

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