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ON THE BOUNDEDNESS OF FUNCTIONS OF (PSEUDO-) DIFFERENTIAL OPERATORS ON COMPACT MANIFOLDS

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1. Introduction. Let M be a compact boundaryless manifold of dimension $n \geqslant 2$, and suppose that P is a first order elliptic (classical) pseudo-differential operator which is positive and self-adjointed with respect to a C^∞ density dx. Then $L^2(M) = L^2(M, dx)$ admits the spectral decomposition

$$L^2(M) = \sum_{j=1}^{\infty} E_j,$$

where the E_j are the eigenspaces corresponding to eigenvalues, λ_j, which can be ordered

$$0 < \lambda_1 \leqslant \lambda_2 \leqslant \cdots.$$

If e_j denotes the projection onto the jth eigenspace, E_j, then any L^2 function can be written as

$$f = \sum_{j=1}^{\infty} e_j(f)$$

where the partial sums converge in L^2, and, moreover,

$$\|f\|_{L^2(M)}^2 = \sum \|e_j(f)\|_{L^2(M)}^2. \tag{1.1}$$

Given a bounded function $m(\lambda)$ we can define operators, $m(P)$, by

$$m(P)f = \sum_{j=1}^{\infty} m(\lambda_j)e_j(f).$$

By (1.1) such operators are always bounded on L^2; however, if one considers any other L^p space, it is known that some smoothness assumptions on the function $m(\lambda)$ are needed to ensure that

$$m(P): L^p(M) \to L^p(M). \tag{1.2}$$

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