Vol. 59, No. 3 DUKE MATHEMATICAL JOURNAL © December 1989

CURVES ON GENERIC KUMMER VARIETIES

GIAN PIETRO PIROLA

Introduction. In this paper we deal with curves of small geometric genus on Kummer varieties. In section 1 we prove a rigidity theorem (see theorem 1). Let C be a curve of genus g lying in the Kummer variety of a q-dimensional Abelian variety. Assuming that the Abelian variety is generated by the inverse image of the curve, theorem 1 states that we have rigidity if $g < q - 1$. The prototype of this result is the fact that a Kummer surface has a global holomorphic (2, 0) form, so it cannot be covered by rational curves. The proof relies on an elementary, but very interesting, lemma of Xiao (cf. [4]). In section 2 we prove a nonexistence theorem in the hypothesis of generality of the Kummer variety for $g < q - 2$. Here we degenerate to Kummer varieties of nonsimple Abelian varieties and use theorem 1. Section 2 can be seen as a method of transforming a rigidity theorem into a nonexistence one. The most surprising consequence is the fact that a generic Abelian variety of dimension $\geqslant 3$ does not contain hyperelliptic curves of any genus. In section 3 we give some examples. We work over the field of complex numbers.

Section 1. Let A be an Abelian variety of dimension $q > 1$, $K = K(A)$ the Kummer variety of A, and let $\pi: A \to K(A)$ be the quotient map. Let C be a smooth curve of genus g, and $\varphi: C \to K$ a nonconstant morphism. We assume that $\pi^{-1}(\varphi(C))$ *generates A as a group* (this is automatic if A is a simple Abelian variety).

We will say that (C, φ) is rigid if the image in K of any deformation of (C, φ) is contained in $\varphi(C)$. If φ is birational onto its image this means that $\varphi(C)$ cannot be deformed in K as curve of *geometric* genus g.

THEOREM 1. *If* $g < q - 1$, (C, φ) *is rigid in* K.

Proof. If (C, φ) is not rigid there exist data (S, B, p, b, σ) where:
—S is a smooth analytic surface,
—B is a smooth analytic curve,
—$p: S \to B$ is a proper smooth morphism,
—b is a point of B such that $p^{-1}(b) \cong C$,
—$\sigma: S \to K$ is a map whose restriction to $p^{-1}(b)$ is the map φ, and such that the image of σ *has dimension* 2.

Received July 27, 1988. Revision received March 17, 1989. This research has been done within the framework of the MPI national project "Geometria Algebrica."