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ON THE LINEAR REPRESENTATION OF BRAID GROUPS, II

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1. Introduction. In [4], it was shown that the Burau and Gassner representations of the braid and pure braid groups arise in a very natural way via a description of these groups as diffeomorphisms acting on representation varieties. The motivation for doing this is that it gives a somewhat broader picture than had been previously available for attacking the problems concerned with these representations.

The focus of this paper is the beginning of an attempt to understand the restrictions on the diffeomorphisms which can occur in the image. This will be done by using the Lie structure; in particular a major role is played by the Campbell—Baker—Hausdorff formula, and some of the methods used in its proof.

More precisely, the program is the following: An element of the braid group defines an element of the diffeomorphism group of the representation variety of a free group. It was shown in [4] that the linearization of this element gives the Burau matrix for this element of the braid group. We now examine the diffeomorphism at levels deeper than the first order. We do this by lifting up the diffeomorphisms in the image of the braid group to nonlinear maps acting on the Lie algebra of the representation variety, and then analyzing the nature of the higher order terms and of the differential equations satisfied by the resulting functions. This is done by using the so-called Campbell—Baker—Hausdorff formula (which is the crucial ingredient in showing that the Lie algebra structure determines the multiplication locally in the group) and methods used in its proof. In fact, this program makes sense even in the case of three strands where these representations are moderately tractable, since the traditional methods often rely on specialization of the variable and perhaps supply more knowledge than understanding.

Some restrictions arise trivially from the special nature of the action of the braid group and the action on the tangent spaces of these invariant manifolds have certain special properties. For example, we have a collection of invariant submanifolds, the trace manifolds (defined in [2]) which carry the Burau representation. For these we can show:

THEOREM 3.3. *If a diffeomorphism lies in the kernel of the Burau representation, then it is the identity to second order on the trace manifolds.*

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