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INTERACTIONS INVOLVING GLIDING RAYS IN BOUNDARY PROBLEMS FOR SEMILINEAR WAVE EQUATIONS

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1. Introduction. This paper is part of a series ([9], [10]) devoted to the study of how singularities spread at the boundary in mixed problems for semilinear wave equations. [10] examined interactions between singularity-bearing *reflecting* rays via arguments that made strong use of this transversality hypothesis. Here we analyze the mechanism of spreading and determine the strength of anomalous singularities for interactions involving at least one gliding ray. For the reader's convenience we have collected in section 2 some results from [9], [10] that are needed in this paper. For background on spreading in free space the reader may consult M. Beals [1], [2].

For $n \geqslant 2$ let $R^n_+ = \{(x, y): x > 0\}$, and set $\Omega = (-\infty, \infty) \times \bar{R}^n_+, \Omega_T = (-T, T) \times \bar{R}^n_+$, and $b\Omega_T = \Omega_T \cap \{x = 0\}$. Consider a function $u(t, x, y) \in H^s_{\text{loc}}(\Omega_T)$, $s > (n + 2)/2$, which satisfies

$$(1.1) \qquad \begin{aligned} &\text{a. } \Box u = (D_t^2 - \Delta)u = \beta(t)f(u) \\ &\text{b. } u|_{b\Omega_T} = 0, \end{aligned}$$

where $\beta(t) \in C_0^\infty$ and $\operatorname{supp} \beta \subset \{|t| < \delta\}$, $\delta < T$. It follows from Theorem 1.3 of [3] that u can have anomalous singularities (i.e., microlocal singularities not present in the solution to the corresponding linear problem: $\Box v = 0$, $v|_{b\Omega_T} = 0$, $v = u$ in $t < -\delta$) of strength at most $\sim 2s - n/2$ (see also [4]). Theorem 1.9 of [10] shows that singularities of this strength can in fact appear when singularity-bearing reflecting rays cross or self-spread at a point on the boundary. This is in contrast to the situation in free space, where Beals's $3s$ theorem [2] implies that a function $u \in H^s_{\text{loc}}$ satisfying (1.1a) can have anomalous singularities of strength at most $\sim 3s - n$.

Suppose now that $u \in H^s_{\text{loc}}(\Omega_T)$ as in (1.1) is smooth in $t < -\delta$, $x > 0$ but has singularities in $t < -\delta$, $x = 0$ along gliding rays. One can ask if it is possible for singularities to appear in $t > -\delta$, $x > 0$ as a result of interactions between gliding rays, and if so, how strong the new singularities are. A special property (discussed below) of the plane tangent to the light cone along any direction corresponding to

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