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GENERALIZED ALBANESE VARIETIES FOR SURFACES IN CHARACTERISTIC $p > 0$

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In this paper we prove the existence of generalized Albanese varieties for smooth proper surfaces in characteristic $p > 0$. Letting X be such a surface and m be a modulus defined by K-theoretic methods in [2], we denote by Cat_m the category of all rational maps $\alpha: X \to G$ into smooth commutative algebraic groups G which admit m as modulus (Definition 3). Our main result is

THEOREM 1. *If α is in Cat_m, then $\alpha^*(\Omega_G^{inv}) \subset H^0(X, \Omega_X(m))$.*

Then by standard arguments we obtain

COROLLARY 2. *In Cat_m there exists $\alpha_{um}: X \to G_{um}$ with the property that any α in Cat_m factors uniquely through α_{um}.*

In [4] we proved these results in characteristic zero in a stronger form, namely, α is in Cat_m if and only if $\alpha^*(\Omega_G^{inv}) \subset H^0(X, \Omega_X(m))$. We also note that Theorem 1 as stated here corrects a remark we made in our announcement [3].

In the course of the proof we will give new proofs of some results used in [4]; these new proofs have the advantage of being valid in arbitrary characteristic. It will also be clear that the methods and the results of this paper and of [4] are valid for smooth projective varieties of arbitrary dimension provided one proves the existence of moduli for rational maps in this generality.

Throughout the paper k is an algebraically closed field of characteristic $p > 0$. X is a proper smooth surface over k with function field K.

For all K-theoretic notation and results we refer to [1] and [2], but we use multiplicative notation.

All algebraic groups are commutative and connected. G_m (resp. G_a, resp. W_n) denotes the multiplicative group (resp. the additive group, resp. the group of Witt vectors of length n).

We recall the following result ([2], ch. 3, Prop. 1):

Let $\alpha: X \to G$ be a rational map into an algebraic group with domain U. Then α induces a homomorphism $\gamma_{m,\alpha}: C_m(X) \to G(k)$ characterized by the property that for all x in U, $1 \in Z = C_m(x)$ is mapped to $\alpha(x)$ by $\gamma_{m,\alpha}$. Here m is a divisor supported on $X \sim U$ and $C_m(X)$ denotes the K-theoretic idéle class group of X with modulus m ([2], ch. 2, §4).

Definition 3. In the above situation we say that α admits m as modulus, and we let Cat_m denote the category of all rational maps admitting m as modulus.

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