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SHARP POLYNOMIAL BOUNDS ON THE NUMBER OF SCATTERING POLES

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1. Introduction. This note is concerned with scattering by a compactly supported bounded measurable potential in $\mathbf{R}^n$, $V \in L_0^\infty$, for the Schrödinger operator $H = \Delta + V$, with $H_0 = \Delta = -\Sigma_{i=1}^n \partial_{x_i}^2$. One standard method to define the scattering matrix, which also carries the physical intuition, is through the wave operators. The wave operators are defined as the following limits:

$$W_\pm = \lim_{t \to \pm\infty} e^{itH_0} e^{-itH}$$

(see, e.g., [3], ch. 14).

The scattering operator, S, relating the free incoming data to the asymptotics of the outgoing perturbed solution is given as

$$S = W_+^* W_- . \tag{1}$$

Since S commutes with the free operator H_0, we can use the spectral decomposition of H_0 in $\mathbf{R}^n$, n odd,

$$H_0 = \int_{-\infty}^{\infty} \lambda^2 \, dE_\lambda ,$$

to write

$$S = \int_{-\infty}^{\infty} S(\lambda) \, dE_\lambda , \tag{2}$$

where now $S(\lambda)$: $L^2(S^{n-1}) \to L^2(S^{n-1})$. This operator is called the scattering matrix. It extends to a family of bounded operators on $L^2(S^{n-1})$ depending meromorphically on $\lambda \in \mathbf{C}$.

The poles of the scattering matrix, known also as resonances, have a long tradition in physics and, for compactly supported potentials (and obstacles), were first rigorously defined by Lax and Phillips [5] (see also Shenk and Thoe [16]). In our convention there are only finitely many poles in the upper half-plane. For real potentials the poles in the upper half-plane all lie on the positive imaginary axis and correspond to the eigenvalues of the Schrödinger operator.

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