Vol. 58, No. 3 DUKE MATHEMATICAL JOURNAL © June 1989

L^p-ESTIMATES ON FUNCTIONS OF THE LAPLACE OPERATOR

MICHAEL E. TAYLOR

0. Introduction. Our goal is to study L^p-continuity of certain functions of the Laplace operator on a complete Riemannian manifold with bounded geometry. To be specific, let M be a complete Riemannian manifold of dimension n. We make the following hypotheses on M:

$$M \text{ has injectivity radius} \geqslant 2\sigma > 0, \tag{0.1}$$

$$M \text{ has } C^\infty \text{ bounded geometry}. \tag{0.2}$$

As is well known, (0.2) implies an exponential bound on the volume growth of balls. We assume a bound of the following form, where $\langle r \rangle^\mu = (1 + r^2)^{(1/2)\mu}$:

$$\operatorname{vol} B_p(r) \leqslant C \langle r \rangle^\mu e^{\kappa r} \tag{0.3}$$

for some $\kappa \geqslant 0$, for the volume of a ball $B_p(r)$ of radius r centered at a point $p \in M$. For example, (0.3) holds, with $\mu = n$, whenever $\operatorname{Ric}_M \geqslant -(n-1)\kappa^2$; some refinements of this can be found in §4 of [4]. Let Δ be the Laplace operator on M; we assume

$$\operatorname{spec}(-\Delta) \subset [A, \infty) \qquad \text{on } L^2(M) \tag{0.4}$$

for some $A \geqslant 0$, and set

$$H = -\Delta - A, \qquad L = H^{1/2}. \tag{0.5}$$

Then, for a continuous function f, $f(L)$ is defined by the spectral theorem. If f is bounded, then $f(L)$ is bounded on $L^2(M)$.

The main result of this paper is a sharp version of a sufficient condition for $f(L)$ to be bounded on $L^p(M)$, derived in Theorem 3.4 of [4]. To state the result, we denote

$$\bar{\Omega}_W = \{\lambda \in \mathbb{C} : |\operatorname{Im} \lambda| \leqslant W\}, \tag{0.6}$$

Received September 16, 1988. Revision received December 8, 1988. Research supported by NSF grant DMS85-02475.