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POINCARÉ SERIES FOR $GL(3, \mathbf{R})$-WHITTAKER FUNCTIONS

ERIC STADE

CONTENTS

Introduction. C. L. Siegel first suggested the construction of a Poincaré series for $GL(2, \mathbf{R})$ that is both automorphic for $GL(2, \mathbf{Z})$ and an eigenfunction of the Laplacian but is not an Eisenstein series. Such a series is built from the $GL(2, \mathbf{R})$ "Whittaker function" $M_\nu(z)$ and, in its domain of convergence, defines a holomorphic function of the complex variable ν.

The properties of Siegel's function were originally studied by Neunhöffer [11] and Niebur [12]. They determined the necessary criteria for convergence and then obtained, among other results, a meromorphic continuation and functional equation for the series in the variable ν. Specifically, this equation states that a certain linear combination of these Poincaré series is equal to the $GL(2, \mathbf{Z})$-Eisenstein series $E(z; \nu)$.

The theory of Neunhöffer and Niebur has recently been extended by Miatello and Wallach [10] to general rank-one groups. On the other hand, when one tries to generalize this theory to $GL(n, \mathbf{R})$ ($n > 2$), a fundamental obstruction occurs, namely, the relevant Poincaré series do not ever converge! It does not even seem possible to define these series as distributions.

Nevertheless it *is* possible, at least in the case of $GL(3, \mathbf{R})$, to give a meaning to Siegel's construction as the analytic continuation and specialization (with respect to a new variable μ) of a linear functional defined on any finite-dimensional space of cusp forms. In this context there then exist a meromorphic continuation and a functional equation analogous to those obtained on $GL(2, \mathbf{R})$. We wish, in this paper, to develop completely the $GL(3, \mathbf{R})$ theory.

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