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VARIATIONS OF HODGE STRUCTURE OF MAXIMAL DIMENSION

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1. Introduction. A variation of Hodge structure is a holomorphic map with values in a Griffiths period domain which satisfies the differential equation

$$\partial F^p/\partial z_i \subset F^{p-1}. \tag{1.1}$$

The purpose of this paper is to give a general (and sharp) bound on the rank of such mappings. That a bound exists is clear from general principles. Equation (1.1) defines a subbundle T^h of the holomorphic tangent bundle of the period domain to which the image of a variation f is tangent, and its fiber dimension gives a first bound on the rank of f [8]. In general, however, the distribution defined by the horizontal tangent bundle T^h is nonintegrable, so that additional restrictions must hold. This is the case whenever D is not of hermitian type. In the simplest case (weight two with $h^{2,0} > 1$) one has the result of [1, 5]:

$$\text{rank } df \leqslant \tfrac{1}{2} \dim T^h, \tag{1.2}$$

or, more explicitly,

$$\text{rank } df \leqslant \tfrac{1}{2} h^{2,0} h^{1,1}. \tag{1.3}$$

The general bound is similar to this: it is given by a piecewise quadratic function of the Hodge numbers for domains of fixed Lie type.

To give a precise statement, fix a period domain D which classifies structures of weight w, let h^q stand for $h^{p,q}$, and set

$$\begin{aligned} m &= [w/2] \\ m^* &= [(w-1)/2] \\ d^i &= h^i h^{i+1} \quad \text{for } i < m^* \\ d^{m^*} &= \tfrac{1}{2} h^{m^*}(h^{m^*}+1) \quad \text{for } w \text{ odd (Type } C) \\ d^{m^*} &= h^{m^*}[h^{m^*+1}/2] + \varepsilon \text{ for } w \text{ even, (Types } B, D), \end{aligned} \tag{1.4}$$

where $\varepsilon = 0$ if $h^{m,m} = h^{m^*+1}$ is even (Type D), $\varepsilon = 1$ if $h^{m,m}$ is odd (type B), and where $h^{m^*} \neq 1$. When $h^{m^*} = 1$, set $d^{m^*} = h^{m^*+1}$.

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