Vol. 58, No. 3 DUKE MATHEMATICAL JOURNAL © June 1989

INEQUALITY FOR THE DISTORTION FUNCTION OF INVERTIBLE SHEAVES ON ABELIAN VARIETIES

SHANYU JI

1. Introduction.

1.1. Let $X = V/L$ be an abelian variety, where L is a lattice in a complex vector space V with $\dim X = g$. Let $\mathscr{L} = \mathscr{L}(\alpha, H)$ be an ample invertible sheaf on X, where the pair (α, H) is the Appel-Humbert data.

The Hermitian metric $(\ ,\ )_1$ on $\mathscr{L}$ is defined by

$$(f, g)_1(x) = e^{-\pi H(x,x)} f(\tilde{x})\overline{g(\tilde{x})}$$

for any $x \in X$, where $\tilde{x}$ is a lifting of x in V, and for any two holomorphic sections $f, g \in \Gamma(X, \mathscr{L})$. A holomorphic section $f \in \Gamma(X, \mathscr{L})$ is identified to be an entire holomorphic function f on V such that

$$f(u + 1) = \alpha(1) e^{\pi H(u,1) + (\pi/2)H(1,1)} f(u)$$

for all $u \in V$ and $1 \in L$. We denote $\|f\|_1^2 = (f, f)_1$. The inner product on $\Gamma(X, \mathscr{L})$ then is defined by $(f, g) = \int_X (f, g)_1 \, d\mu$, where $d\mu$ is the Harr measure on X such that $\int_X d\mu = 1$. We also denote $\|f\|^2 = (f, f)$.

The Fubini-Study metric on $\mathscr{L}$ is defined by

$$(f, g)_2 = \sum_i (f, \varphi_i)_1(x)\overline{(g, \varphi_i)_1(x)},$$

where $f, g, \varphi_i \in \Gamma(X, \mathscr{L})$, and φ_i, $i = 1, 2, \ldots, \dim \Gamma(X, \mathscr{L})$, is an orthonormal basis of $\Gamma(X, \mathscr{L})$ with respect to $(\ ,\ )$. The definition of $(\ ,\ )_2$ is independent of the choices of the orthonormal basis. Note that $(f, g)_2 = (\Sigma_i (\varphi_i, \varphi_i)_1(x))(f, g)_1$.

To measure the distortion between these two metrics, we introduce a positive real analytic function $b_{\mathscr{L}(1)}(x)(\ ,\ )_1(x) = (\ ,\ )_2(x)$, for all $x \in X$, i.e., $b_{\mathscr{L}(1)}(x) = \Sigma_i (\varphi_i, \varphi_i)(x)$.

1.2. It was proved by Kempf [2] that for any X, $\mathscr{L}$ as above, there are positive constant numbers C_1 and C_2 such that

$$(\#) \qquad C_1 m^{2g} \leqslant b_{\mathscr{L}(m^2)}(x) \leqslant C_2 m^{2g}$$

Received September 26, 1988. Revision received December 15, 1988.