Vol. 58, No. 3 DUKE MATHEMATICAL JOURNAL © June 1989

ALGEBRAIC VARIETIES PRESERVED BY GENERIC FLOWS

VALERY LUNTS

1. Introduction. Let $\mathbf{A}_n$ be the Weyl algebra of linear differential operators with polynomial coefficients on $\mathbf{C}^n$. The algebra $\mathbf{A}_n$ is generated by y_i, ∂_{y_i}, $i = 1, \ldots, n$, which stand for multiplication by the variable y_i and differentiation with respect to y_i. Recall the following filtration of $\mathbf{A}_n$:

$$F^0 \subset F^1 \subset \cdots,$$

where F^k is spanned by monomials in $\{y_i, \partial_{y_i}\}$ of total degree $\leqslant k$.

Let $D \in \mathbf{A}_n$ be such that $D \in F^{k+1}$, $D \notin F^k$. Then the *symbol* $\sigma(D)$ is the image of D in F^{k+1}/F^k. Put $\sigma(y_i) = x_i$, $\sigma(\partial_{y_i}) = x_{i+n}$. Then the associated graded algebra $\operatorname{gr} \mathbf{A}_n = \bigoplus F^{k+1}/F^k$ is isomorphic to the commutative polynomial ring $\mathbf{C}[x_1, \ldots, x_{2n}]$.

Choose $D \in \mathbf{A}_n$ and consider the cyclic left $\mathbf{A}_n$-module $M = \mathbf{A}_n/\mathbf{A}_n D$. Our main result is that for "almost all" D's the module M is irreducible. If $n \geqslant 2$, then M is not holonomic. Hence "most" irreducible $\mathbf{A}_n$-modules are not holonomic.

Let us explain the expression "for almost all" D's.

Denote by Σ^k the space of homogeneous polynomials in the x_i of degree k. We say that a property S holds for a *generic* P in Σ^k if the set $\{P \in \Sigma^k | S \text{ does not hold for } P\}$ is contained in a countable number of hypersurfaces in Σ^k.

Let $k \geqslant 4$. Consider the following property S of polynomials $P \in \Sigma^k$: For any $D \in \mathbf{A}_n$ such that $\sigma(D) = P$, the left ideal $\mathbf{A}_n D$ is maximal. We prove that S holds for a generic $P \in \Sigma^k$.

In [BL] it was shown how the above algebraic problem can be solved in geometric terms. Let us recall the main idea.

The space $\mathbf{C}^{2n}$ is a symplectic manifold with the symplectic form

$$\omega = \Sigma_{i=1}^n dx_i \wedge dx_{i+n}.$$

To every function P on $\mathbf{C}^{2n}$ there is associated its Hamiltonian vector field

$$h_P = \Sigma_{i=1}^n \left(\frac{\partial P}{\partial x_{i+n}}\right)\partial_{x_i} - \left(\frac{\partial P}{\partial x_i}\right)\partial_{x_{i+n}}.$$

Definition. We say that a vector field ξ on $\mathbf{C}^{2n}$ *preserves* a subvariety $Y \subset \mathbf{C}^{2n}$ if ξ_p is tangent to Y at every smooth point $p \in Y$.

Received July 11, 1988, Revision received November 16, 1988.