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# THE ROCKLAND CONDITION FOR NONDIFFERENTIAL CONVOLUTION OPERATORS

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**Introduction.**

1. In their joint paper [16], Hulanicki and Jenkins apply methods of harmonic analysis on nilpotent Lie groups in order to achieve results concerning the summability of eigenfunction expansions of Schrödinger operators on $\mathbb{R}^n$ whose potentials are sums of squares of polynomials. They also obtain Marcinkiewicz-type multiplier theorems for such opertors. Their basic idea is to regard the Schrödinger operator $L$ under consideration as $L = \pi_{\mathscr{L}}$, where $\mathscr{L} = \Sigma X_j^2$ is a sublaplacian on an appropriately chosen stratified nilpotent Lie group $N$, and $\pi$ is an irreducible unitary representation of $N$. The $C^\infty$-regularity of the heat kernel for $L$ plays a very essential role.

A recent paper of the same authors [17] is devoted to the investigation of the Schrödinger operator

$$L = -d^2/dx^2 + p(x) \tag{1}$$

on the line, where $p$ is a polynomial of degree at least 1. The idea behind their approach is similar to that described above. All other tools and ingredients of their theory being ready, the paper is devoted to proving the existence and $L^2$-regularity of the densities of a continuous stable semigroup of measures on a pretty simple nilpotent Lie group of a possibly very high step of nilpotence. The infinitesimal generator of this semigroup $P$, which is a convolution with a homogeneous distribution on $N$, now takes over the role of the sublaplacian.

The argument of Hulanicki and Jenkins, based partially on techniques of Fefferman and Phong [5], is complicated, and it is essentially restricted to the particular form (1) of their one-dimensional operator. However, as the authors point out, their method could be applied to, e.g., multidimensional Schrödinger operators

$$L = -\sum \partial^2/\partial x_j^2 + V(x),$$

where

$$V(x) = \sum_{k=1}^{m} |p_k(x)|^{a_k}, \qquad a_k > 0,$$

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