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CHARACTERISTIC CLASSES OF PRINCIPAL BUNDLES IN ALGEBRAIC INTERSECTION THEORY

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Introduction. The following notation will be maintained throughout this paper:

k is an algebraically closed field.

A *scheme* is a scheme of finite type over k.

If Γ is an algebraic group over k, by a *principal* Γ*-bundle* we mean a Γ-bundle in the category of schemes which is locally trivial in the étale topology.

G is a connected affine algebraic group over k.

T is a maximal torus of G.

The integer n is the dimension of T.

B is a Borel subgroup of G containing T.

U is the unipotent radical of B.

W is the Weyl group of G with respect to T.

$\hat{T}$ is the group of characters of T.

$\operatorname{Sym}(\hat{T})$ is the symmetric algebra of $\hat{T} \otimes \mathbf{Q}$ over $\mathbf{Q}$.

$\operatorname{Sym}(\hat{T})^W$ is the algebra of invariants under the natural action of W on $\operatorname{Sym}(\hat{T})$.

I is the ideal of $\operatorname{Sym}(\hat{T})$ generated by homogeneous elements of positive degree which are invariant under the action of W.

Λ is the ring $\operatorname{Sym}(\hat{T})/I$.

Assume that $k = \mathbf{C}$. Then the following results, due to Leray and Borel, are well known in topology.

Let BG be the classifying space of G. Then there is a natural isomorphism

$$\operatorname{Sym}(\hat{T})^W \cong H^*(BG, \mathbf{Q}).$$

Consequently, for each topological principal G-bundle $E \to M$ we get a characteristic homomorphism of graded $\mathbf{Q}$-algebras

$$c_E\colon \operatorname{Sym}(\hat{T})^W \to H^*(M, \mathbf{Q}).$$

If

$$\alpha_E\colon \operatorname{Sym}(\hat{T}) \to H^*(E/B, \mathbf{Q})$$

is the homomorphism that sends a character t of T to the first Chern class of the

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