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A CLASSIFICATION OF INVARIANT KNOTS

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0. Introduction. This paper presents a classification of high-dimensional simple knots of odd dimension, which are left invariant under a free $\mathbf{Z}_m$-action on the ambient sphere. A knot is *simple* if its complement has the homotopy type of S^1 up to, but not including, its middle dimension. Note that if this holds for one more dimension the knot is trivial [L1]. By [L-S] and [L3], a simple knot is determined by the homotopy type of its complement. Recall [M2] that algebraic knots are always simple.

Simple knots of odd dimensions have been classified up to isotopy by Levine [L3] in terms of s-equivalence of Seifert matrices. The isotopy classification of even-dimensional simple knots was analyzed by Kearton [Kt1], [Kt2], and by Farber [F].

In this paper we consider triples $(J^{2n+1}, K^{2n-1}; T)$, where (J, K) is a simple PL or smooth knot, $n \geqslant 2$, and T is a free $\mathbf{Z}_m$-action on the ambient sphere J leaving K invariant. Such triples, with (J, K) not necessarily simple, have been classified up to equivariant concordance by S. Cappell and J. Shaneson [C-S] in terms of Γ-groups and by N. Stoltzfus [St1], [St2] in terms of Seifert linking forms and isometric structures.

Two invariant knots are said to be *equivariantly homeomorphic* if there exists an action-preserving, orientation-preserving (PL, DIFF) self-homeomorphism of the ambient sphere, sending one knot to the other. In the nonequivariant case this is the same as isotopy. The first half of this paper presents a classification of simple invariant knots up to equivariant homeomorphism. This classification is done in terms of Seifert linking forms.

A Seifert surface for a knot (J, K) is an oriented, codimension-one submanifold of J whose boundary is the knot K. In the equivariant category it is also required that the translates of the Seifert manifold are disjoint from each other, except along their boundary. The Seifert pairing is defined on the middle-dimension homology of this manifold. In the nonequivariant case there is only one such form for each Seifert manifold V. In the equivariant case there is a collection $\{B_i | i = 0, \ldots, m-1\}$ of linking forms [St2] defined on $H_n(V)$ by

$$B_i(x, y) = L(x, v_+ T^i y).$$

Here L is the linking in the ambient sphere and v_+ is an ε-push in the positive normal direction.

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