Vol. 58, No. 1 DUKE MATHEMATICAL JOURNAL © February 1989

ALGEBRAIC CYCLES ON NONSIMPLE ABELIAN VARIETIES

FUMIO HAZAMA

§0. Introduction. In this paper we investigate the algebraic cycles on the product $A \times B$ of abelian varieties A, B defined over $\mathbb{C}$. Generally, even if we can determine the structure of the Hodge ring or the ring of algebraic cycles of each factor, it is not so easy to do the same for their product. The purpose of the present paper is to show that this is possible in some cases. More precisely, we will show the following:

THEOREM 0.1. *Let A, B be abelian varieites defined over $\mathbb{C}$. Assume that they are stably nondegenerate (see (1.1) for the definition) and that they have no simple components of type* IV *(see* [6]*). Then the product $A \times B$ is also stably nondegenerate, in particular, the Hodge conjecture holds for it.*

In some sense, this theorem reduces the investigation of Hodge cycles on abelian varieties to the study of those on simple abelian varieties as far as one deals with types I, II, and III. (As for the reason why we must exclude the type IV, see §3.) As a corollary of this, we will show that the product $J_1(M) \times J_1(N)$ of the jacobian varieties of the modular curves $X_1(M)$, $X_1(N)$ satisfies the Hodge conjecture (see (2.5)).

I express here my gratitude to Professor Shuji Yamagata for valuable conversation. Also, I would like to thank Ms. Michiko Toki for her careful reading of the manuscript.

Notation. For a smooth projective variety X defined over $\mathbb{C}$, we denote by $\mathscr{B}^*(X)$ the Hodge ring $\bigoplus_{i=0}^{\dim X} \mathscr{B}^i(X) = \bigoplus_{i=0}^{\dim X} H^{2i}(X, \mathbb{Q}) \cap H^{i,i}(X)$ and by $\mathscr{D}^*(X)$ the subring of $\mathscr{B}^*(X)$ generated by divisor classes.

Let G (resp. $\mathfrak{g}$) be a group (resp. Lie algebra) and let V be a vector space with G-action (resp. $\mathfrak{g}$-action). Then we denote by $\mathrm{End}_G\, V$ (resp. $\mathrm{End}_{\mathfrak{g}}\, V$) the space of G-linear (resp. $\mathfrak{g}$-linear) endomorphisms of V. We denote by $[V]^G$ (resp. $[V]^{\mathfrak{g}}$) the space of G-invariant (resp. $\mathfrak{g}$-invariant) elements in V. For an abelian variety A, $Hg(A)$ denotes the Hodge group of A (see [7] for the definition), and we write $L(A)$ for the Lie algebra of $Hg(A)_{\mathbb{C}}$. Finally, we recall the definition of the *reduced dimension* of an abelian variety. When A is a simple abelian variety,

Received February 6, 1988.