Vol. 58, No. 1 DUKE MATHEMATICAL JOURNAL © February 1989

TRANSFERENCE PRINCIPLES FOR RADIAL MULTIPLIERS

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1. Introduction. Let T be a bounded linear operator in $L^2(\mathbb{R}^n)$ which is invariant under the action of the group $M(n)$ of motions in $\mathbb{R}^n$. Such an operator can be represented as

$$Tf(x) = T_m f(x) = \int_{\mathbb{R}^n} \hat{f}(\xi) m(|\xi|) e^{2\pi i x \circ \xi}\, d\xi,$$

where $\hat{f}$ denotes the Fourier transform of f and $m \in L^\infty(\mathbb{R}_+)$. The L^p theory of such operators has been extensively investigated (see [2], [5], [6], [10], [11], [14], [22], [24]), but our understanding of the problem is still rather incomplete, especially in dimensions $n > 2$. In this paper we obtain estimates for operators of the above form and variants of them (e.g., maximal operators or square functions with the same invariance properties) in spaces different from $L^p(\mathbb{R}^n)$, namely:

(a) Mixed norm spaces $L^p(L^2)(\mathbb{R}^n) = L^p_{\text{rad}}(L^2_{\text{ang}})$, where the L^2 norm is taken with respect to the angular variables and the L^p norm with respect to the radial variable

(b) Weighted L^2 spaces:

$$L^2(w) = L^2(\mathbb{R}^n, w(x)\, dx) = \left\{ f\colon \int |f(x)|^2 w(x)\, dx < \infty \right\},$$

especially for radial weights of the form $w(x) = |x|^\alpha$.

The main result of type (a) is Theorem 3.1, which says that the analogue of the Bochner-Riesz conjecture, with L^p replaced by $L^p(L^2)$, is true in all even dimensions. This means that the spherical summation operator T^λ defined as above with $m(t) = m_\lambda(t) = (1 - t^2)^\lambda_+$, $0 < \lambda \leqslant (n-1)/2$, is bounded in $L^p(L^2)(R^n)$ for $2n/(n + 1 + 2\lambda) < p < 2n/(n - 1 - 2\lambda)$, n even.

On the other hand, we obtain in section 4 a radial Littlewood-Paley and multiplier theory in $L^2(\mathbb{R}^n, |x|^\alpha\, dx)$ without regularity assumptions. The negative answer given by C. Fefferman [11] to the disc conjecture means that there is no L^p theory ($p \neq 2$) for irregular radial multipliers; weighted L^2 spaces present themselves as a reasonable alternative, which was already explored by Hirschman [16] before the disc conjecture was solved. In Theorem 4.2, an analogue of the Littlewood-Paley

Received April 4, 1988.