Vol. 57, No. 3 DUKE MATHEMATICAL JOURNAL © December 1988

ON POSITIVE SOLUTIONS OF SECOND-ORDER ELLIPTIC EQUATIONS, STABILITY RESULTS, AND CLASSIFICATION

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0. Introduction. Let P be a second-order uniformly elliptic operator defined in a domain $\Omega \subseteq \mathbf{R}^n$. Let $\mathscr{C}_P(\Omega)$ be the convex cone of all positive solutions of the equation

$$Pu = 0 \quad \text{in } \Omega. \tag{0.1}$$

If one assumes that P or Ω satisfies some very special properties, e.g., that P and Ω are invariant under rotations or translations, then one can get an explicit description of the structure of $\mathscr{C}_P(\Omega)$ and also delicate perturbation results (see, for example, [2], [4], [8], [9], [11]).

In this paper our aim is different. We wish to discuss some *general* properties of $\mathscr{C}_P(\Omega)$ for *general* P and Ω and to obtain *some* information on the structure of $\mathscr{C}_P(\Omega)$. The paper is divided into four sections. In section 1 we shall give some basic definitions, fix notations, and recall briefly some known results. In section 2 we shall compare the structure of $\mathscr{C}_{P_i}(\Omega)$, $i = 1, 2$, where P_2 is a "small" perturbation of P_1. It turns out that in the general case a "small" perturbation is a change of the coefficients of P_1 in a compact set in Ω. We shall see in section 3 that under some assumptions the analogue for a small perturbation for the case $\Omega = \mathbf{R}^n$ is a perturbation of the form $P_2 = P_1 + W$, where $W \in L^1(\mathbf{R}^n)$. In section 4 we shall study the question of stability and instability of $\mathscr{C}_P(\Omega)$ by investigating the behavior of $\mathscr{C}_{P+tW}(\Omega)$ when $t \in \mathbf{R}$ is varied and W is a fixed function.

We shall confine ourselves to classical solutions. The results are also valid for weak and strong solutions; the proofs differ only in minor details from the proofs for the classical solutions. Some of the results in this paper were announced first in [9]. I would like to remark that similar results were obtained by M. Murata for Schrödinger operators ([8]; see also [3]).

Acknowledgements. The paper is based on part of the author's Ph.D. thesis, completed in 1986 at the Hebrew University of Jerusalem [10]. I wish to express my deep gratitude to my thesis advisor, Professor Shmuel Agmon, for the encouragement, support, and help he gave me.

Received October 22, 1987.