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A VARIATIONAL APPROACH TO THE EXISTENCE OF COMPLETE EMBEDDED MINIMAL SURFACES

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In [11], we established the existence of a sequence of properly embedded minimal surfaces, M_k, of genus k, with finite total curvature and three ends, $k \geqslant 1$. The surface M_1 in this sequence is the example of Costa [6], [9]. Later, other examples of properly embedded minimal surfaces of finite total curvature were shown to exist using similar analytic methods [4], [10].

In spite of the power of these analytic techniques, they have not been successful in answering basic theoretical and qualitative questions about embedded minimal surfaces of finite total curvature with a large number of ends. The present paper is an attempt to address such questions and, at the same time, to develop a general variational approach to proving the existence of embedded minimal surfaces of finite total curvature in $\mathbb{R}^3$.

Section 1 contains technical results on the existence of embedded compact minimal surfaces with large symmetry. In Section 2, we provide a construction for the surfaces M_k; they arise from a generating sequence of compact minimal surfaces by the blowing up of a singularity in the limit. Actually, we construct by this method three-ended, properly embedded minimal surfaces of finite topology with a large symmetry group. It then follows from Statement 8 of the Main Theorem, proved in [11] and stated below, that these surfaces must be the surfaces M_k. This alternative construction suggests that there should be other, less symmetric, examples.

In [3] and [5], we apply the results of this paper to construct new examples of periodic minimal surfaces that have an infinite number of annular ends.

Recently Pitts and Rubenstein [20] have also given a variational construction of complete minimal surfaces $\{N_k\}$ of genus k with three ends. Their construction is similar in spirit to our variational approach. Their examples have the same symmetry as the examples $\{M_k\}$ and hence, by the Main Theorem below, are the same surfaces.

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For completeness we restate the characterization of the examples $\{M_k\}$ from [11].

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